Securitisation and Optimal Foreclosure*

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Abstract

Does securitisation distort the foreclosure decision of non-performing mortgages? In a model in which an informed securitiser *jointly* designs the mortgage-backed security and the foreclosure policy, we find that the securitiser with high-quality pool optimally adopts an *excessive* foreclosure policy and sells a risky debt (the senior tranche) to uninformed investors. Foreclosure effectively mitigates the adverse selection friction in securitisation by making the risky debt *less* information sensitive. Our model predicts that foreclosure likelihood, loan loss, mortgage servicers' capacity and incentive to modify delinquent loans endogenously vary with the quality of the underlying mortgage pool. Policies that aim to restore ex post efficient foreclosures may inadvertently reduce mortgage originators' screening effort. (*JEL* D8, G21, G23, G24)

Keywords. Security design, mortgage-backed securities, mortgage foreclosure, mortgage servicers, asymmetric infor mation, commitment

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1 Introduction

In the aftermath of the subprime mortgage crisis, there are over 14 million U.S. properties with foreclosure filings from 2008 to 2014.[1] Particularly, privately securitised mortgages account for more than half of these foreclosures. This raises concerns by academics and policymakers over whether securitisation has aggravated the crisis by hindering the modification of delinquent mortgages.[2] Consequently, the U.S. government has developed the Home Affordable Modification Program (HAMP) to incentivise mortgage modification instead of foreclosure.[3]

However, understanding the foreclosure of securitised mortgages and the effect of such policy interventions has been hindered by the absence of a theoretical framework that allows one to answer some fundamental questions. How does securitisation affect the securitising banks' (the securitisers) choice of foreclosure policy in equilibrium? Do public interventions in foreclosure such as HAMP carry further implications for the origination and the securitisation of mortgages?

We develop a model of asset-backed securitisation with *endogenous* foreclosure. In our model, as in DeMarzo (2005), an informed securitiser would like to design and sell a security backed by the cash flow from her mortgage pool to uninformed investors. We depart from DeMarzo (2005) by allowing the securitiser to also choose a policy of foreclosing or modifying any fraction of the delinquent mortgages in the future.[4] Foreclosure policy in our model affects the mortgage pool's cash flow because foreclosure reduces the pool's exposure to borrow re-default risk. Specifically, foreclosing a delinquent mortgage and selling the underlying property provides a safe cash flow, whereas modification delivers a higher (lower) cash flow when the once-defaulted borrower recovers (re-defaults).[5] A securitiser who wants

[1]Source: RealtyTrac (2015).

[2]See, for example, Posner and Zingales (2009),Piskorski et al. (2010), Agarwal et al. (2011), Adelino et al. (2013) and Kruger (2016).

[3]HAMP provides direct one-off and annual monetary incentives to mortgage servicers for each successfully modified delinquent mortgage. For a detailed description and an empirical evaluation of HAMP, see Agarwal et al. (2012).

[4]In practice, foreclosure or modification is often carried out by a separate entity known as the servicer, who is hired by the securitiser and is compensated according to the mortgage servicing contract. Our baseline model applies to the extent that the securitisers can influence the foreclosure policy. This can be achieved by choosing servicers of different modification capacity and/or by tailoring the foreclosure incentive components in the mortgage servicing contracts. In section 4, we extend the model to explicitly account for the role of servicers in implementing foreclosure policies.

[5]For simplicity, we do not distinguish modification from forbearance, i.e. simply continuing the

to maximise the expected mortgage pool value can choose an efficient foreclosure policy, under which she forecloses to the point that the marginal foreclosure proceeds equal the marginal expected proceeds from modification.

The main finding of the paper is that adverse selection problem in securitisation causes distortions in foreclosure policy. When there is heterogeneity in the quality of the underlying mortgage pool privately observed by the securitiser, the securitiser with a higher-quality, less default-prone mortgage pool (the high type) optimally sells a risky debt (the senior tranche) to uninformed investors. Intuitively, debt is least sensitive to the private information held by the securitiser. Equivalently, the securitiser signals her quality to investors via costly retention of the equity tranche.[6,7] The new insight of our model is that foreclosure further reduces the information sensitivity of a debt security. Therefore, the optimal policy features *ex post excessive* foreclosure. That is, a securitiser may wish to foreclose a delinquent mortgage even if modification would have delivered a higher expected loan value.

The benefit of foreclosure in our model is to reduce the optimal debt security's information sensitivity. This contrasts with the standard "money burning" intuition. In fact, both excessive and *insufficient* foreclosure policies are burning money and are more costly for the low-quality mortgage pool. In our framework, in contrast, the optimal foreclosure policy is excessive precisely because the optimal security is concave (risky debt). We illustrate that if the security is restricted to be linear (unlevered equity) or convex (levered equity), the optimal foreclosure policy can be biased *against* foreclosure. This underlies an important message of the paper, namely the *direction* of the distortion in the foreclosure policy caused by

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mortgage contract with the defaulted borrowers. As it will soon be clear, there is no qualitative difference in the interpretation of our model.

[6]The notion of debt as the optimal security due to its information insensitivity dates back to the Pecking Order Theory in Myers and Majluf (1984). DeMarzo et al. (2015) shows that in an ex-post liquidity-based security design game like ours, standard debt is the least information sensitive, thus optimal monotone security when the cash flow satisfies Hazard Rate Ordering (HRO) property. See also Chemla and Hennessy (2014) and Vanasco (ming) for recent theoretical works with costly retention of equity tranche as signals. Empirically, Begley and Purnanandam (2017) find that conditional on observable characteristics, RMBS deals with larger equity tranche have lower delinquency rate and command higher prices, suggesting that information asymmetry is relevant and the signalling mechanism is at play.

[7]As also discussed in Begley and Purnanandam (2017), even if the securitiser sells off the equity tranche at a later date, the initial retention of equity tranche could still be a costly signal because i) the opportunity cost of the locked-up capital could still be significant in a high-growth mortgage market and ii) the equity tranches in practice are often sold to sophisticated and informed investors like hedge funds and mutual fund managers, who are likely to have stronger bargaining position and/or scarcer capital than uninformed senior tranche investors.

securitsation depends on the endogenous security design.

Why then does excessive foreclosure reduce the information sensitivity of the optimal debt security? To answer this, let's consider the low-type securitiser's mimicking incentive. When the low-type securitiser chooses the same security and foreclosure policy as the high type does, she earns a mispricing premium from securitisation by selling the security at a higher price than its true value, because the investors wrongly believe that it is backed by a high-quality pool. The crucial observation is that when foreclosure increases, the cash flow of the pool becomes safer and hence the senior tranche, a concave security, becomes more valuable due to Jensen's inequality. Furthermore, this increase in value due to foreclosure is *larger* for the senior tranche backed by the *low-quality* mortgage pool which defaults more often. The senior tranche thus becomes less information sensitive, i.e. less rewarding for the low type to mimic. Altogether, foreclosure allows the high type to securitise more in equilibrium. To the extent that security design concerns the distribution of cash flow rights between owner and investors and that foreclosure policy affects the distribution of total asset cash flow, the insight from the paper would apply to the *joint* determination of a firm's financing and (dis-)investment policy in general.[8]

Our model sheds light on various empirical and institutional aspects of the mortgage industry. Our main result yields predictions for observed foreclosure rates of securitised mortgages consistent with empirical findings. First, foreclosure of securitised mortgages is excessive, as shown by Maturana (2016).[9] Second, securitisation of the mortgages leads to higher foreclosure rates conditional on delinquency. Piskorski et al. (2010), Agarwal et al. (2011), and Kruger (2016) have shown that, compared to similar mortgages held in banks' portfolios, securitised mortgages are more likely to be foreclosed conditional on delinquency.[10] Our model generates consistent predictions because for bank-held mortgages, the bank does not

[8]For example, an informed owner of a start-up firm who privately knows the probability of reaching some performance-based milestones should jointly design the optimal structure of securities to be sold to investors and the subsequent investment or liquidation plan when the milestones are met or not.

[9]Maturana (2016) finds that an exogenous increase in marginal modification, hence a decrease in foreclosure, of non-agency securitised mortgage reduces 40% of loan loss, suggesting that foreclosure is value-destroying at the margin.

[10]Using an earlier sample from 2005 to 2007 and a different methodology, Adelino et al. (2013) do not find statistically different probability of modification between securitised and portfolio loans. See Agarwal et al. (2011) and Kruger (2016) for a discussion on the different findings of Adelino et al. (2013).

face an adverse selection problem. Third, our theory predicts that the increase in foreclosure caused by securitisation is concentrated among high-qualiy securisters. Indeed, Piskorski et al. (2010) and Agarwal et al. (2011) highlight the substantial heterogeneity in the effects of securitisation on foreclosure rates, with larger effects among higher-quality loans.

Our results can reconcile with the apparent inefficiencies in the mortgage servicers' contracts, which are repeatedly cited as an impediment to efficient foreclosure.[11] In practice, mortgage servicers are responsible for making the foreclosure or modification decision of delinquent mortgages. Empirically, there is heterogeneity in their capacity and incentives to conduct loan modification, resulting in economically significant variations in modification rates.[12] In our model, excessive foreclosure stems from the securitisers' incentives to mitigate information friction and requires the securitisers' commitment power. This suggests an economic rationale for the securitiser with a high-quality pool to hire an external mortgage servicer to implement an excessive foreclosure policy. In Section 4, we show that the securitiser can do so by either choosing a servicer with limited capacity to modify loans (or known to be "tough"), or providing the servicer with a compensation contract that is biased towards foreclosure.[13] The former setup implies a novel endogenous matching mechanism between mortgage pools' quality and servicers' modification capacity while the latter predicts endogenous biases in the servicing contracts towards foreclosure, which is documented by Levitin and Goodman (2009), Thompson (2009) and Kruger (2016).

In the last part of the paper, we extend the model to endogenise the securitiser's ex ante screening effort choice at origination and derive policy implications for regulating foreclosures. Our message is a cautionary one in the Lucas' critique fashion: policies aiming to restore ex post efficient foreclosure such as HAMP

[11]See, for example, Posner and Zingales (2009), Piskorski et al. (2010) and Kruger (2016). HAMP is also a policy response directly targeted at the servicers' incentives towards foreclosure.

[12]Agarwal et al. (2011) and Agarwal et al. (2012) find economically significant servicer fixed effects in predicting modification rate, which they attribute to the heterogeneity in servicer-specific capacity in modification. Agarwal et al. (2012) and Maturana (2016) find that changes in servicers' incentives, due to HAMP and policy changes in the GSE, have significant impact on modification rates of delinquent mortgages. See also Kruger (2016) for an analysis of servicers' discretion and compensations in prime MBS deals.

[13]Note that it is plausible that MBS investors do infer information from the identity of the servicer and/or the mortgage servicing contract, since such information is provided in the servicing contract, or the Pooling and Servicing Agreement (PSA), alongside the prospectus of the MBS issue.

would inadvertently *reduce* the securitiser's incentive to screen mortgages diligently, leading to lower average quality of the mortgage pools and overall welfare in the economy. When a securitiser with a high-quality mortgage pool can no longer signal her quality effectively with an excessive foreclosure policy, her initial incentive to exert screening effort in order to form a high-quality mortgage pool is weakened. Although information asymmetry in securitisation always results in under-provision of the securitisers' screening effort, ex post excessive foreclosure in our model is a remedy, instead of a symptom, of the problem. This analysis suggests that, while HAMP is a temporary measure to reduce foreclosure, a longer-term policy should take into account its effects on the origination and securitisation of mortgages.

Our model starts from a discrete cash flow version of models of liquidity-based security design, such as DeMarzo and Duffie (1999), DeMarzo (2005) and Biais and Mariotti (2005). We depart from the literature by allowing the securitisers to take actions that affect the distribution of the underlying asset's cash flow. In the context of mortgage securitisation, foreclosure policy of delinquent mortgages is one of such actions. While excessive foreclosure emerges as an additional dimension of the signal in equilibrium, the optimal security in our model is debt, consistent with the classical literature on security design with a privately informed issuer, started with Myers and Majluf (1984) and Nachman and Noe (1994).

Several papers have highlighted the incentive problems associated with securitisation. In a setting of securitisation under adverse selection similar to ours, Chemla and Hennessy (2014) and Vanasco (ming) analyse how liquidity in the MBS market affects ex ante loan originators' screening effort. Hartman–Glaser et al. (2012) and Malamud et al. (2013) also study the optimal design of the originator's compensation contracts to incentivise screening effort in a dynamic setting. These papers do not study the foreclosure policy of the delinquent mortgages. In contrast, we first characterise the optimal foreclosure policy and then assess its effect on the originator's screening incentives.

Our results contribute to the understanding of the role of servicers and their incentive contracts. Mooradian and Pichler (2014) study the asset composition (pooling) of the mortgage pool and show that a non-diversified pool alleviates the servicer's moral hazard problem. Our paper instead focuses on the securitisation (tranching) problem under asymmetric information for a mortgage pool of given quality and shows that even in the absence of principle-agent frictions, it is *ex ante* optimal to have an *ex post* inefficient foreclosure policy.

Our paper also relates to but differs from the literature on optimal loan modification and foreclosure policy. Wang et al. (2002) and Riddiough and Wyatt (1994) argue that borrowers' strategic default incentives lead lenders to adopt a tough foreclosure policy in order to deter non-distressed borrowers from opportunistic behaviour. However, these papers do not analyse the securitisation of the loans and their models based on borrower strategic default do not readily explain the difference in the foreclosure rates between securitised and portfolio loans. Gertner and Scharfstein (1991) focuses on the free-riding problem among multiple creditors. Our paper highlights that information asymmetry in securitisation can be another important factor that determines foreclosure decisions.

The rest of the paper is organised as follows. Section 2 describes the model setup. Section 3 carries out the main analysis of the equilibrium with endogenous foreclosure policy. Section 4 highlights the importance of commitment over the foreclosure policy and discusses the role of mortgage servicers in enabling such commitment power. Section 5 extends the model to consider ex ante screening incentives of the securitiser in relation to the subsequent foreclosure policy. Section 6 discusses the empirical implications of the model. Section 7 concludes.

2 Model setup

This section sets up the model and comments on the assumptions which are central to the model.

There are four dates: 0, 1, 2 and 3. The model's participants consist of a bank and a continuum of outside investors. The main analysis of this paper (Section 3–4) concerns only $t = 1, 2, 3$. We extend the model to an ex ante stage $t = 0$ only in Section 5.

All agents are risk neutral. The outside investors are deep-pocketed and competitive. The banks are impatient and have a discount factor $\delta < 1$ between $t = 1$ and $t = 3$. This follows the assumption of DeMarzo and Duffie (1999) and can be interpreted as the bank's liquidity needs. The outside investors have no such discount. Hence, there are gains from trade between the bank and the investors.[14]

Mortgage pool and foreclosures

The underlying asset in our model is a mortgage pool containing a continuum of ex ante identical mortgages that pay off at $t = 3$. For the main analysis, we focus on the foreclosure of the mortgages when they become delinquent, as detailed below. In Section 5 we extend the model to consider the ex ante screening effort choice, which also affects the cash flow of the mortgage pool.

We model the mortgage pool as a well-diversified portfolio of mortgages. The mortgage pool is thus only exposed to aggregate risks, which affect the ability for all borrowers to repay.[15] Specifically, with probability π, the mortgage pool is in a good state (G) and no borrowers default. In state G, the mortgage pool returns a riskless cash flow $Z_G > 0$. With probability $1 - \pi$, the mortgage pool is in a bad state (B) and each borrower defaults with some i.i.d. probability. Thanks to the diversification benefit, the proportion of the mortgages that become delinquent at $t = 2$ is fixed. We normalise the measure of the delinquent mortgages in the pool to 1. The remaining performing mortgages continue to return a riskless cash flow $Z_B < Z_G$ at $t = 3$. Since mortgage delinquency only occurs in the bad state, we will focus primarily on the sequence of events after a realisation of the bad state to study mortgage foreclosures.

When a mortgage becomes delinquent at $t = 2$, it can be foreclosed or modified.[16] In the case of foreclosure, the mortgage contract is terminated. The collateral property is repossessed and sold to outside investors. Alternatively, if the delinquent mortgage is modified, the modified mortgage pays off a riskless cash flow $X > 0$ with probability θ at $t = 3$ (recovery) or zero otherwise (re-default). For simplicity, we assume that the recovery (and re-default) of delinquent mortgages in a give pool are perfectly correlated. This is also in line with the assumption of a well-diversified

[14]Modelling gains from trade as a discount factor $\delta < 1$ is standard in the literature to capture liquidity needs stemming from, e.g., capital constraints, new investment opportunities, risk-sharing, etc. (see Holmström and Tirole (2011)).

[15]Such aggregate risks can be aggregate property prices or employment opportunities for the borrowers.

[16]Throughout the paper, we use "mortgage modification", "mortgage renegotiation", and "mortgage forbearance" interchangeably. Because we abstract from the renegotiation process between the mortgage lender and the borrower, one can interpret the cash flows to the mortgage pool following a decision of no foreclosure as the un-modelled optimal renegotiation outcome.

mortgage pool so that only aggregate risks affect the repayment of the borrowers. Finally, we further assume that $Z_G \geq Z_B + X$, so that the value of a mortgage in the good state is at least as high as in a bad state, even if all delinquent mortgages are modified and subsequently resume payments in the bad state. Intuitively, the difference accounts for the value of the temporary missing payments and the modified lower principal and interest repayment.

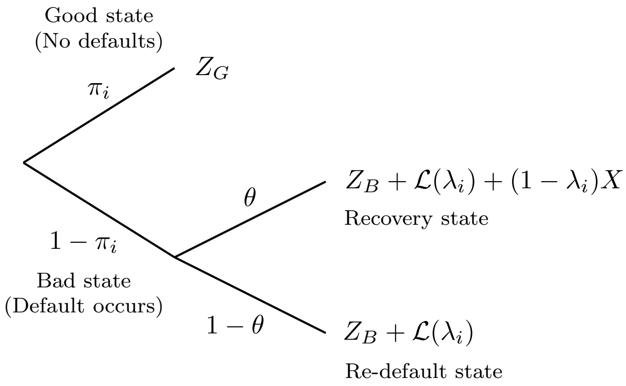
The mortgage pool's exposure to the aggregate risks is characterised by the probability of entering state G. This probability $\pi \in \{\pi_H, \pi_L\}$, where $\pi_H > \pi_L$, is mortgage-pool specific and is the source of information asymmetry between the bank and outside investors, as detailed in the next section. We interpret π_i as the "quality" of the mortgage pool (subscript "H" stands for "High" and "L" for "Low"). A high-quality pool is less exposed or more resilient to the aggregate risks and hence is more likely to have no delinquent mortgages (be in the good state G). At $t = 1$, all model participants have the prior belief that $\pi = \pi_H$ with probability γ.[17] One interpretation of γ is a publicly observable signal of the quality of the loans in a mortgage pool, e.g. the average FICO scores of the borrowers. Therefore, a pool with higher γ is an observably better because it is more likely to be a high-quality pool that defaults less often.

The focus of the paper is to study what proportion of the delinquent mortgages is chosen to be foreclosed in equilibrium and how securitisation affects this decision. The foreclosure policy can be summarised by $\lambda_i \in (0, 1)$, the fraction of delinquent mortgages foreclosed in a mortgage pool of quality i (i.e. $(1 - \lambda_i)$ fraction of delinquency mortgages modified). Denote by $\mathcal{L}(\lambda_i)$ the total liquidation proceeds from repossessed properties. For a given foreclosure policy, the overall cash flow from a type i mortgage pool at $t = 3$ is then Z_G with probability π_i (the "Good" state), $Z_B + \mathcal{L}(\lambda_i) + (1 - \lambda_i)X$ with probability $(1 - \pi_i)\theta$ (the "Recovery" state), and $Z_B + \mathcal{L}(\lambda_i)$ with probability $(1 - \pi_i)(1 - \theta)$ (the "Re-default" state), as illustrated in Fig 1.

The exact functional form of the liquidation proceeds $\mathcal{L}(\lambda)$ depends on the characterisation of the market for distressed properties as well as the direct and indirect costs associated with foreclosures. We abstract from these considerations

[17]In Section 5 we endogenise this probability γ in an ex ante stage $t = 0$ through the bank's screening effort choice.

Figure 1: Mortgage pool cash flow



to keep the analysis general and make the following intuitive assumption on the foreclosure technology.

Assumption 1. *For* $\lambda \in (0, 1)$*, (i)* $\mathcal{L}(\lambda)$ *is strictly increasing and concave; (ii)* $\frac{\partial \mathcal{L}(\lambda)}{\partial \lambda} \in (0, X)$*; and (iii)* $\lim_{\lambda \to 0^-} \frac{\partial \mathcal{L}}{\partial \lambda} > \theta X > \lim_{\lambda \to 1^+} \frac{\partial \mathcal{L}}{\partial \lambda}$.

Assumption 1 states that, first, $\mathcal{L}(\lambda)$ is strictly increasing and concave in λ. The decreasing marginal liquidation value of the foreclosed loans could be due to un-modelled heterogeneity in the market value of the underlying properties at $t = 2$. All else equal, a delinquent mortgage backed by a property of higher market value would be foreclosed first. Alternatively, the decreasing marginal liquidation value of the foreclosed loans could also be due to either scarce capital or scarce expertise in making the renovation needed to realise the value of the properties. Second, the marginal liquidation value of the mortgage is below the full repayment value of the mortgage $\frac{\partial \mathcal{L}(\lambda)}{\partial \lambda} \in (0, X)$ for any $\lambda \in (0, 1)$. Intuitively, there are costs associated with liquidating a mortgage, due to, for example, renovation and repair costs associated with investing in distressed property, as well as other outstanding liabilities such as unpaid fees and taxes. The last part of this assumption is a technical assumption to ensure an interior optimal foreclosure policy in the first-best case.

Securitisation

Because of the liquidity discount δ, at $t = 1$, the bank who owns the mortgage pool would like to design and sell a security backed by the cash flow of the mortgage pool at $t = 3$ to outside investors. We will henceforth refer to the bank as the "securitiser" and the security as the mortgage-backed securities (MBS). The securitiser thus receives the cash proceeds from selling the MBS at $t = 1$, and retains any residual cash flow from the mortgage pool after paying off the investors at $t = 3$.

We mentioned earlier that there is asymmetric information between the securitiser and the investors. This creates frictions in the securitisation process akin to the classical lemon's problem in Akerlof (1970). Specifically, at the beginning of $t = 1$, the securitiser receives private information regarding the quality of the mortage pool $\pi_i \in \{\pi_H, \pi_L\}$. The source of private information could come from new information produced during the process of structuring the individual mortgages into a pool for securitisation, as in DeMarzo and Duffie (1999).[18]

The focus of this paper is the foreclosure decision, which interacts with the securitisation process because the foreclosure policy affects the cash flows of the mortgage pool. We model the securitisation process as follows. The securitiser with information π_i offers the outside investors a security $\mathcal{F}_i$ and promises a foreclosure policy λ_i. The security $\mathcal{F}_i$ is contracted upon the cash flows at $t = 3$, specifying payments to the MBS investors for each realisation of the cash flow. We restrict our attention to monotone securities.[19]

We assume that the securitiser is able to commit to the foreclosure policy promised at $t = 1$ and then implement it as $t = 2$, when mortgage defaults occur. The securitiser's ability to commit is crucial to our main results. We show how

[18] DeMarzo and Duffie (1999) solves the ex ante security design problem, whereas we solve for the ex post security design problem after the banks learn about their private information. As shown by DeMarzo (2005) and DeMarzo et al. (2015), similar intuition carries through in the ex post problem, although the problem becomes more complicated as the design itself becomes a signal.

[19] That is, a higher realisation of the mortgage pool cash flow should leave both the outside investors and the securitiser a (weakly) higher payoff. Although this implies some loss of generality, it is not uncommon in the security design literature, e.g. Innes (1990) and Nachman and Noe (1994). One potential justification provided by DeMarzo and Duffie (1999) is that, the issuer has the incentive to contribute additional funds to the assets if the security payoff is not increasing in the cash flow. Similarly, the issuers has the incentive to abscond from the mortgage pool if the security leaves the issuer a payoff that is not increasing in the cash flow.

the results change if the securitiser did not have commitment power and discuss how the securitiser can establish commitment power over her foreclosure policy by contracting with a mortgage servicer in Section 4.

After observing the offer $(\mathcal{F}_i, \lambda_i)$, the competitive investors form a posterior belief $\hat{\pi}$ regarding the private information of the securitiser, and bid the price of the security p to its fair value. At $t = 3$, after paying investors according to $\mathcal{F}_i$ from the mortgage pool cash flow, the securitiser consumes any residual cash flow.

Time line and the equilibrium concept

The timeline of the model is summarised in Table 1. The main analysis carried out in Section 3–4 concerns only $t = 1, 2, 3$. We extend the model to an ex ante stage $t = 0$ in Section 5.

Table 1: Model timeline

$t = 0$	Securitiser exerts screening effort (Section 5 only)
$t = 1$	Securitiser observes π_i and offers $(\mathcal{F}_i, \lambda_i)$
$t = 2$	Mortgage defaults in state B Securitiser implements foreclosure policy λ_i
$t = 3$	Final payoffs realise

The equilibrium concept in this model is the perfect Bayesian equilibrium (PBE). Formally, a PBE consists of a security $\mathcal{F}_i$ issued by the securitiser of each type $i \in \{H, L\}$, the foreclosure policy λ_i of the securitiser of each type, and a system of beliefs such that i) the securitiser chooses the security and the foreclosure policy at $t = 1$ to maximise her expected payoff, given the equilibrium choices of the other agents and the equilibrium beliefs, and ii) the beliefs are rational given the equilibrium choices of the agents and are formed using Bayes' rule (whenever applicable). As there can be multiple equilibria in games of asymmetric information, we invoke the Intuitive Criterion of Cho and Kreps (1987) to eliminate equilibria with unreasonable out-of-equilibrium beliefs. This allows us to restrict attention to only the least cost separating equilibrium (as shown in Lemma 2).

Discussion of the framework

We adopt and extend the liquidity-based security design framework developed by DeMarzo (2005) to study the joint optimisation problem of securitisation and foreclosure under asymmetric information. As we shall later illustrate, our mechanism involves the retention of equity tranche as a signal of quality and requires the securitisers to be able to promise and commit to an ex post excessive foreclosure policy. In Section 4, we will introduce mortgage servicers in the model and argue that the securitisers could implement the promised foreclosure policy by contracting with third-party mortgage servicers. We discuss in this section below the plausibility of using equity tranche as a signal in practice.

Securitisers in our model design and sell securities to raise as much liquidity as possible as in DeMarzo (2005). Similarly, the optimal MBS in our model is risky debt. In other words, securitisers with high-quality mortgage pools signal information to investors through the retention of equity tranche. While the contribution of the paper concerns the role of foreclosure policy in securitisation, we believe this framework of securitisation captures realistic aspects of the MBS market and the signalling mechanism is supported by empirical evidence.[20] For example, Begley and Purnanandam (2017) find that, conditional on observable characteristics, RMBS deals with larger equity tranches have lower delinquency rates and command higher prices, suggesting that investors could and do learn from the equity tranche size.

While it is possible that in practice the retained tranche might be subsequently sold off in the secondary market, the *initial* retention of equity tranche could still signal information as long as there are substantial (opportunity) costs associated with it. We believe that this is likely to be true for two reasons.

First, the delayed sales of the equity tranche could be costly to the securitisers because it implies that some capital is locked-up and thus the securitisers have to forgo some profitable lending in the mortgage market or investments in general. In addition, mortgages could default during this retention period, rendering the securitister in a less desirable position to sell the equity tranche of a pool with defaulted mortgages. Indeed, the warehouse risk could be a substantial concern and was discussed extensively when the subprime mortgage crisis emerged. For a

[20]See the discussion in footnote 6 for recent theoretical works featuring the same mechanism and for supporting evidence in the RMBS market.

model and evidence of delayed loan sales as a costly signal for loan quality in the mortgage market, see Adelino et al. (2016).

Second, due to the segmentation of markets for different MBS tranches, the securitisers are also likely to be in a less advantageous position in the sales of equity tranche than those of the senior tranche. As discussed in detail in Begley and Purnanandam (2017), the typical buyers in the senior tranche are regulated institutions such as insurance companies who primarily trade for regulatory reasons. On the other hand, the equity tranches are most often sold to hedge fund mangers who trade primarily for profit and are likely to be have more expertise in valuing complex mortgage securities. As a result, securitisers with low-quality pools would face stronger market discipline in the equity tranche market and thus incur a relatively larger cost. Under this interpretation, the securitisers incur a cost of the initial retention of equity tranche which is either the liquidity discount δ if she decides to retain the equity tranche or the part of the gains from securitisation conceded to the equity tranche investors due to weaker bargaining position.

3 Securitisation with endogenous foreclosure

In this section we analyse our model of securitisation with endogenous foreclosure. In order to contrast with the distortion in the foreclosure policy created by information asymmetry, we first present the benchmark case under full information, before we proceed to solve the model under asymmetric information.

3.1 First-best and the full-information benchmark

In this section we first characterise the first-best foreclosure policy. We then analyse the benchmark equilibrium under full information, and show that the first best is achieved in the full-information equilibrium.

The first-best foreclosure policy maximises the value of the mortgage pool $V_i(\lambda)$.

$$\lambda_i^{FB} = \arg\max_{\lambda} \; V_i(\lambda) \tag{1}$$

$$\text{where} \quad V_i(\lambda) \equiv \pi_i Z_G + (1-\pi_i)[Z_B + \mathcal{L}(\lambda) + (1-\lambda)\theta X] \tag{2}$$

The solution is characterised by the first order condition, $\frac{\partial \mathcal{L}(\lambda_i^{FB})}{\partial \lambda} = \theta X$. That is, since the marginal value obtained from foreclosure is decreasing with the fraction of foreclosed loans, the first-best level of foreclosure is determined such that the the margin value from foreclosure is equal to the expected recovery value given modification, conditional on the bad state (B). Furthermore, as the H and L type mortgage pools only differ in the probability of entering state (B), the first-best level of foreclosure is identical across types. Denote the solution to this first order condition by $\lambda^{FB} \in (0, 1)$.

We now characterise the equilibrium under full information. Firstly consider the optimal security issued in the securitisation process at $t = 1$. Since any retention of the cash flows by the securitiser incurs a liquidity discount, it is optimal for the securitiser to sell the entire cash flow of the mortgage pool to the investors, when all securities are fairly priced given full information. Secondly, given that the entire cash flow is securitised, the securitiser optimally commits to the first-best foreclosure policy λ^{FB} to maximise the value of the mortgage pool and hence her payoff.

The following proposition thus summarises the full-information benchmark results. All proofs are in the appendix.

Lemma 1. *In the full-information benchmark, the securitiser of both types securitises the entire cash flow of the mortgage pool, and chooses the first-best foreclosure policy* λ^{FB}.

Denote henceforth the expected payoff to a type i securitiser in the full-information benchmark by $U_i^{FB} \equiv V_i(\lambda^{FB})$. We would like to conclude this section by stressing the fact that the first-best foreclosure policy is achieved in the full-information benchmark equilibrium. Therefore, any inefficiency in the equilibrium foreclosure policy in this paper is driven by information asymmetry between the securitiser and the outside investors. We shall turn to this asymmetric information problem in the next section.

3.2 Asymmetric information, tranching and excessive foreclosure

In this section, we first show that the optimal security is a risky debt in equilibrium. We then prove our main result that an excessive foreclosure policy is optimal given that the MBS is a debt security.

At $t = 1$, the securitiser with private information π_i issues an MBS $\mathcal{F}_i$ backed by the cash flows of the mortgage pool, and promises a foreclosure policy λ_i. Observing the offer from the securitiser $(\mathcal{F}_i, \lambda_i)$, the investors form a belief about the quality $\hat{\pi}$ of the mortgage pool. We focus our attention on the least cost separating equilibrium, which is the unique equilibrium that satisfies the Intuitive Criterion in our model. We state this result in the following lemma and formally prove it in the appendix.

Lemma 2. *The unique equilibrium that satisfies the Intuitive Criterion is the least cost separating equilibrium.*

Let's now start the analysis with the securitiser who owns a low-quality, more default-prone mortgage pool. In a separating equilibrium, the low-type securitiser always receives the fair price on the security issued. Therefore the securitiser maximises her expected payoff by selling the entire cash flow from the mortgage to outside investors, and promising the first-best level of foreclosure policy. There is no distortion in the form of either inefficient retention or inefficient foreclosure for the low type. Denote by U_i^* the expected payoff to a type i securitiser in equilibrium. The payoff to the low-type securitiser in a separating equilibrium is thus equal to the first-best level, $U_L^* = U_L^{FB}$, while her foreclosure policy in equilibrium is $\lambda_L^* = \lambda^{FB}$. We denote henceforth with superscript $*$ all equilibrium quantities.

The high-type securitiser, on the other hand, has to issue a security and promise a foreclosure policy such that in equilibrium it is not profitable for the low type to deviate and mimic. In order to focus on our main result regarding the optimal foreclosure policy in equilibrium, we establish that the optimal security is a risky debt in the following lemma, while relegating the full characterisation of the security design problem to the appendix.

Lemma 3. *For any pre-committed foreclosure policy λ_H, a risky debt backed by the mortgage pool with face value $F_H \in (Z_B + \mathcal{L}(\lambda_H), Z_G)$ is an optimal security for the high-type securitiser.*

This result is consistent with the classic literature on the pecking order of outside financing, e.g. Myers (1984) under asymmetric information.[21] The optimal security

[21] Technically, the cash flow distribution in our model satisfies the Hazard Rate Ordering (HRO)

issued by the high-type is a debt security, which is the least information sensitive. Moreover, the high-type exhausts her capacity of issuing risk-free debt ($F_H > Z_B + \mathcal{L}(\lambda_H)$), since risk-free securities are free from the information asymmetry problem. The high-type securitiser's retention of residual claims of future cash flow could be seen as a necessary signalling cost in order to separate from the low type, a well-established result in the security design literature such as Leland and Pyle (1977) and DeMarzo and Duffie (1999).

We now proceed to solve for the optimal foreclosure policy, taking as given that the optimal security issued by the high-type securitiser is a risky debt. We will henceforth refer to the debt security sold to outside investors as the "senior tranche", the cash flows retained by the securitiser as the "equity tranche", and the face value of the debt security (denoted by F) as the "size" of the senior tranche. For example, the maximum size of the senior tranche is $F^{max} = Z_G$ and the corresponding size of the equity tranche is $(Z_G - F^{max}) = 0$.

Let us denote by $p_i(F, \lambda)$ the value of the MBS given a face value of F and a foreclosure policy λ, backed by a mortgage pool of quality i. That is,

$$p_i(F, \lambda) = \pi_i F + (1 - \pi_i)[\theta \min\{Z_B + \mathcal{L}(\lambda) + (1 - \lambda)X, F\} + (1 - \theta)(Z_B + \mathcal{L}(\lambda_H))] \tag{3}$$

In a separating equilibrium, after observing (F_H, λ_H), the investors believe that the issuer of the security with face value F_H who promises a foreclosure policy of λ_H is of the high type. The market price of the MBS is therefore equal to $p_H(F_H, \lambda_H)$.

In the least cost separating equilibrium, the high-type securitiser maximises the value of the proceeds from securitisation plus the discounted value of the residual cash flow by choosing the face value of the debt security, F_H, and her promised foreclosure policy λ_H. Her equilibrium payoff is given by

$$\begin{aligned} U_H^* &= max_{(F_H, \lambda_H)} \quad p_H(F_H, \lambda_H) + \delta\left[V_H(\lambda_H) - p_H(F_H, \lambda_H)\right] \\ s.t. \quad (IC) \quad U_L^{FB} &\geq p_H(F_H, \lambda_H) + \delta\left[V_L(\lambda_H) - p_L(F_H, \lambda_H)\right] \end{aligned} \tag{4}$$

where (IC) is the incentive compatibility constraint for the low type not to mimic

property, which is weaker than the Monotone Likelihood Ratio Property (MLRP) commonly assumed in signalling environments. DeMarzo et al. (2015) show that the (HRO) is a sufficient condition to ensure the optimality of debt security in a signalling framework with liquidity needs.

the offer (F_H, λ_H) of the high type. Denote by (F_H^*, λ_H^*) the unique solution to the above optimisation programme.

The following proposition highlights a key property of the equilibrium foreclosure policy, which is the main result of the paper.

Proposition 1. *In the least cost separating equilibrium, the high-type securitiser adopts a (weakly) excessive foreclosure policy in equilibrium, whereas the low-type securitiser adopts the first-best foreclosure policy. That is,*

$$\lambda_H^* \geq \lambda^{FB} = \lambda_L^*$$

The weak inequality is strict if and only if $G(\lambda^{FB}) > 0$*, where*

$$G(\lambda) \equiv \delta\pi_L Z_G + (1-\delta\pi_L)[Z_B + \mathcal{L}(\lambda) + (1-\lambda)X] - (1-\pi_H)(1-\theta)(1-\lambda)X - U_L^{FB} \qquad (5)$$

As shown in the appendix, the condition $G(\lambda^{FB}) > 0$ given by Eq. 5 implies that the equilibrium face value of the debt satisfies $F_H^* \leq Z_B + \mathcal{L}(\lambda_H^*) + (1-\lambda_H^*)X$. That is, the optimal senior tranche is relatively small in size and, importantly, only defaults in the re-default state, but not in the recovery state. Proposition 1 states that, in an equilibrium in which the MBS issued by the high type is relatively safe, the equilibrium foreclosure policy of the high type deviates from the first-best policy and is qualitatively excessive. $G(\lambda^{FB}) > 0$ is more likely to hold when i) the cost of retention is low (high δ) and/or ii) information asymmetry is severe (high π_H and/or low π_L), so that a large equity tranche must be retained by the high type in order to signal her quality.

To highlight the trade-off faced by the high-type securitiser when choosing her foreclosure policy in equilibrium, we can rewrite her expected payoff as follows.

$$\delta V_H(\lambda_H) + (1-\delta)p_H(F_H, \lambda_H) \qquad (6)$$

The high-type securitiser in equilibrium chooses the optimal securitisation and foreclosure policy to maximize the sum of two terms — the discounted value of the mortgage pool enjoyed by the securitiser without securitisation, and the liquidity gains from the sale of the senior tranche.

Then, another way to state our main result in Proposition 1 is that, while excessive foreclosure reduces the value of the mortgage pool, $V_H(\lambda_H)$, it allows the high-type securitiser to realise more gains from securitisation, i.e. higher $p_H(F_H, \lambda_H)$. Denote $\hat{F}_H(\lambda_H)$ as the optimal size of the senior tranche that the high-type securitiser offers in a separating equilibrium, *for a given foreclosure policy* λ_H. Intuitively, since under a fixed λ_H, the value of the senior tranche $p_H(F_H, \lambda_H)$ increases with its size, the high type would offer the largest possible senior tranche that does not trigger mimicry from the low type. That is, the optimal size $\hat{F}_H(\lambda_H)$ is simply the F_H that binds the (IC). Since $\frac{\partial V_H}{\partial \lambda_H}|_{\lambda=\lambda^{FB}} = 0$, showing $\frac{dp_H}{d\lambda_H}|_{\lambda_H=\lambda^{FB}} \geq 0$ is sufficient to establish the (weak) excessiveness of the optimal foreclosure policy. The following corollary summarises the above discussion.

Corollary 1. *In a separating equilibrium, excessive foreclosure allows the high-type securitiser to realise (weakly) higher securitisation proceeds with the optimal senior tranche. That is,*
$\left.\frac{dp_H(\hat{F}(\lambda_H),\lambda_H)}{d\lambda_H}\right|_{\lambda_H=\lambda^{FB}} \geq 0$, *where the inequality is strict if and only if* $G(\lambda^{FB}) > 0$.

It remains to answer the question: Why does excessive foreclosure allow the high type to securitise more? This is because foreclosure further reduces the information sensitivity of the optimal senior tranche, thereby deterring the low type's mimicry effectively. Let's start with a closer look at the low type's no-mimicking, incentive compatibility constraint

$$U_L^{FB} \geq \delta V_L(\lambda_H) + \underbrace{p_H(F_H, \lambda_H) - \delta p_L(F_H, \lambda_H)}_{\text{premium from securitising with } (F_H, \lambda_H)} \tag{7}$$

Relaxing the no-mimicking constraint is equivalent to lowering the low type's mimicking payoff, i.e. the right-hand-side of (Eq. 7), as U_L^{FB} is not affected by the high type's action (F_H, λ_H). The mimicking payoff comprises of two parts – i) the discounted value of the low type's portfolio $\delta V_L(\lambda_H)$ following a deviation to adopt the foreclosure policy of the high type, λ_H, and ii) the premium (in utility units) from securitising under the terms offered by the high type, where the low-type securitiser receives $p_H(F_H, \lambda_H)$ from the investors whilst only giving up a security that is worth $\delta p_L(F_H, \lambda_H)$ to her. We exclusively focus on the premium $p_H(\cdot) - \delta p_L(\cdot)$ in the following discussion because the expected value of the pool V_L is not affected by a marginal deviation from λ^{FB}.

We now show that an excessive foreclosure policy can effectively reduce the low type's mimicking premium $p_H(\cdot) - \delta p_L(\cdot)$ and in turns allows the high type to achieve a larger gain from securitisation. This implies that a marginally excessive foreclosure policy can lower the low-type securitiser's mimicking incentive. This result relies on the following properties of the MBS, which is a senior tranche that prioritises the pay out of cash flows in the worse states. The value of the MBS is more sensitive to an increase in the foreclosure rate, and less sensitive to an increase in the tranche size, when backed by a low-quality mortgage pool than when backed by a high-quality mortgage pool. Specifically,

$$(1-\pi_L)(1-\theta)\,\mathcal{L}'(\lambda) = \frac{\partial p_L(F,\lambda)}{\partial \lambda} > \frac{\partial p_H(F,\lambda)}{\partial \lambda} = (1-\pi_H)(1-\theta)\,\mathcal{L}'(\lambda) \qquad (8)$$

$$1-(1-\pi_L)(1-\theta) = \frac{\partial p_L(F,\lambda)}{\partial F} < \frac{\partial p_H(F,\lambda)}{\partial F} = 1-(1-\pi_H)(1-\theta) \qquad (9)$$

In particular, Eq. 8 gives the main insight of our paper that foreclosure reduces the information sensitivity of the optimal senior tranche: foreclosure reduces the wedge between the senior tranche's valuation with a high-quality pool and that with a low-quality pool.

Then, start from the case in which the high-type securitiser chooses the first-best foreclosure λ^{FB} and the maximum incentive-compatible face value of the MBS $\hat{F}_H(\lambda^{FB})$. Consider a marginal increase in the high-type securitiser's foreclosure policy $\lambda' > \lambda^{FB}$, accompanied by a corresponding decrease in the face value $F' < \hat{F}_H(\lambda^{FB})$, that keeps the value of the MBS $p_H(\cdot)$ unchanged. Because of Eq. 8 and 9, these changes result in an increase in the low type's valuation $p_L(\cdot)$, thereby reducing the mimicking premium $p_H(\cdot)-\delta p_L(\cdot)$ and relaxing the (IC). Finally with a slack (IC), the high type can increase the face value to $\hat{F}_H(\lambda') > F'$ and achieve a strictly higher securtisation proceeds. It is perhaps worth pointing out that while excessive foreclosure enables the high type to securitise more, i.e. to sell a *more valuable* MBS, it does not necessarily lead to a *bigger* senior tranche.[22] In other words, the signalling cost in our model, defined as the high type's loss of utility with respect to the symmetric information benchmark $(U_H^{FB} - U_H^*)$, can be lower even

[22]It can be shown that, excessive foreclosure increases the face value of the MBS the high-type securitiser can issue without triggering mimicry, or $\left.\frac{\partial \hat{F}_H(\lambda_H)}{\partial \lambda_H}\right|_{\lambda_H=\lambda^{FB}} \geq 0$, if and only if $\delta \geq \frac{1-\pi_H}{1-\pi_L}$.

if the retained equity tranche size is larger, because of the endogenous foreclosure policy.

The properties specified by Eq. 8 and 9 are relatively intuitive. Recall that foreclosure effectively transfers cash flow in the mortgage pool from the recovery state to the re-default state. Because the senior tranche MBS only defaults and pays the whole mortgage pool cash flow in the re-default state, and pays face value F_H otherwise, foreclosure increases the expected value of the MBS. Moreover, as an MBS backed by the low-quality mortgage pool defaults more often and pays the face value less often than that backed by the high-quality mortgage pool, an MBS backed by the low-quality mortgage pool is more sensitive to a marginal increase in foreclosure (Eq. 8) and less sensitive to an increase in face value (Eq. 9).

This analysis uncovers the importance of security design in determining the optimal foreclosure policy. We have shown that the high-type securitiser's optimal foreclosure policy is excessive precisely because the optimal MBS is a risky debt. The logic behind this result is different from the idea of committing to a costly action to signal private information. In particular, both excessive and insufficient foreclosure are costly in the sense that they reduces the value of the mortgage pool, and both are costlier for the low-type than for the high-type securitiser as the low type's mortgage pool is more prone to default. However, the properties of the MBS (Eq. 8–9) imply that the opposite distortion towards an overly lenient foreclosure policy, or insufficient foreclosure, is never optimal. We further explore the implications of security design for the optimal foreclosure policy in Section 3.3.

Finally, the following comparative statics further illustrates the tradeoff faced by the high-type securitiser under asymmetric information. An increase in the quality of the high type, π_H, exacerbates the information asymmetry because it creates greater mimicking incentives. As a result, there is more distortion towards excessive foreclosure in equilibrium.

Proposition 2. *The equilibrium foreclosure policy of the high-type securitiser, and hence the distortion in the foreclosure policy, is increasing in the quality of her mortgage pool. That is, $\frac{\partial \lambda_H^*}{\partial \pi_H} \geq 0$, where the inequality is strict if and only if $G(\lambda^{FB}) > 0$.*

3.3 Security design and foreclosure policy

In this section, we highlight the importance of security design in determining the foreclosure policy, by contrasting the optimal foreclosure policy given different security designs. Recall that, Section 3.2 shows that the securitiser optimally chooses an excessive foreclosure policy given that she issues debt, a concave security. While debt is shown to be the optimal security in Lemma 3, in this section, we consider the securitiser's foreclosure decision while restricting the security design of the MBS to be a linear security (i.e. unlevered equity), or a convex security (i.e. retaining the senior debt tranche and issuing the junior equity tranche).

Firstly, suppose the securitiser must issue a linear security, whose payoff is equal to a constant fraction $\alpha \in [0,1]$ of the realised cash flow produced by the mortgage pool. The value of the MBS is thus equal to $\alpha V_i(\lambda)$, if the securitiser is of type i and chooses a foreclosure policy λ. Similar to the analysis in Section 3.2, in any separating equilibrium, the low-type securitiser sells the entire cash flow to outside investors and chooses the first-best foreclosure policy, because she always receives the fair price on her security. The high-type securitiser's optimisation problem, analogous to Eq. 4, can be expressed as

$$\begin{aligned} &\max_{(\alpha,\lambda_H)} \quad \alpha V_H(\lambda_H) + \delta(1-\alpha)V_H(\lambda_H) \\ s.t. \quad (IC) \quad & U_L^{FB} \geq \alpha V_H(\lambda_H) + \delta(1-\alpha)V_L(\lambda_H) \end{aligned} \tag{10}$$

where (IC) is the incentive compatibility constraint for the low type not to mimic the offer (α, λ_H) of the high type.

The following proposition states that, given a linear security, securitisation can distort foreclosure in either direction. We denote the equilibrium quantities in this case by superscript UE, which stands for "unlevered equity".

Proposition 3. *When the MBS is restricted to be a linear security (unlevered equity), in a least cost separating equilibrium, the high-type securitiser's foreclosure policy can be excessive, insufficient, or first-best. The low-type securitiser adopts the first-best foreclosure policy.*

This result in Proposition 3 stands in stark contrast to the optimal foreclosure policy under risky debt, which is always excessive. The optimality of the distortion under linear security follows from the standard "money burning" logic: the high-type securitiser "burns" enough value of the mortgage pool via distortion in foreclosure policy to deter mimicking by the low type. Since either an excessive or an insufficient foreclosure policy can destroy the value of the pool, both can be optimal. In particular, we show in the proof of Proposition 3 that there can exist two levels of equilibrium foreclosure policy, $\lambda_{H,1}^{UE}$ and $\lambda_{H,2}^{UE}$, where $\lambda_{H,1}^{UE} < \lambda^{FB} < \lambda_{H,1}^{UE}$, such that $U_H(\lambda_{H,1}^{UE}) = U_H(\lambda_{H,2}^{UE}) < U_H(\lambda^{FB})$. Figures 2a and 2b illustrate the difference in the distortion in the high-type securitiser's foreclosure policy given the optimal debt security and given a linear security.

We also consider the case where the securitiser retains the senior debt tranche and issues the junior equity tranche. In other words, the MBS is restricted to be a levered equity claim.[23] We demonstrate with a numerical example, illustrated in Figure 2c, that when the MBS is levered equity, the high-type securitiser optimally chooses an overly lenient foreclosure policy. This follows from the same intuition as for the case of debt security: a decrease in foreclosure makes the cash flows from the mortgage pool riskier and thereby raises the MBS's expected value, as the MBS is a convex claim. As before, the MBS backed by the low-quality pool receives a larger increase in value and hence becomes less information sensitive. To sum up, the analysis in Section 3.3 highlights the importance of security design on the specific distortions in foreclosure policy.

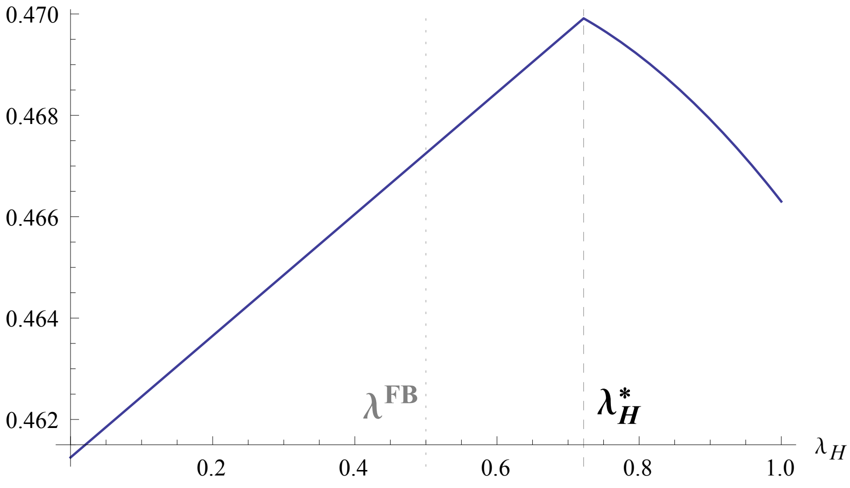
[23] The analysis of the problem with levered equity as the MBS is as follows: in equilibrium, the low-type securitiser sells the entire cash flow and chooses the first-best foreclosure policy, as analysed in Section 3.2. The high-type securitiser must choose the foreclosure policy and the security design to maximise her expected payoff. Let $K \leq Z_G$ denote the face value of the senior debt tranche retained by the securitiser. The high-type securitiser's optimisation problem, analogous to Eq. 4, is given by

$$\begin{aligned} \max_{(K,\lambda_H)} \quad & p_H(K,\lambda_H) + \delta\left[V_H(\lambda_H) - p_H(K,\lambda_H)\right] \\ s.t. \quad (IC) \quad & U_L^{FB} \geq p_H(K,\lambda_H) + \delta\left[V_L(\lambda_H) - p_L(K,\lambda_H)\right] \end{aligned}$$

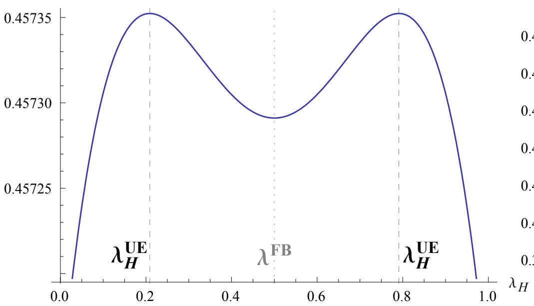
where $p_i(K,\lambda)$ is the value of the MBS (the junior equity tranche) given a face value of the senior debt tranche K and a foreclosure policy λ, given by

$$\begin{aligned} p_i(K,\lambda) = \pi_i(Z_G - K) + (1-\pi_i)\theta \max\{Z_B + \mathcal{L}(\lambda) + (1-\lambda)X - K, 0\} \\ + (1-\pi_i)(1-\theta)\max\{Z_B + \mathcal{L}(\lambda) - K, 0\} \end{aligned}$$

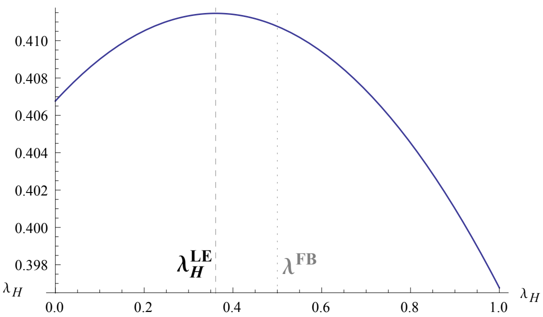
Figure 2: Expected payoff of the high-type securitiser as a function of the foreclosure policy λ_H given different security designs. We use the binding (IC) to eliminate the security design variables F_H, α and K for Panels (a), (b) and (c) respectively, and plot the resulting expected payoff of the high-type securitiser as a univariate function of λ_H. The parameter values used in this plot are: $Z_B = 0.2$, $Z_G = 0.55$, $X = 0.3$, $\theta = 0.5$, $\pi_H = 0.8$, $\pi_L = 0.5$, $\delta = 0.8$, and $L(\lambda) = \frac{1-(1-\lambda)^2}{2}X$. Given these parameter values, $\lambda^{FB} = 0.5$, $\lambda^*_H = 0.72$, $\lambda^{UE}_H = \{0.21, 0.79\}$ and $\lambda^{LE}_H = 0.36$.



(a) Optimal security (Debt, concave security)



(b) Linear security



(c) Convex security

4 Commitment and mortgage servicers

Thus far we have made the important assumption that the securitiser is able to commit to a foreclosure policy at $t = 1$ and then implement it at $t = 2$ when default occurs. In this section, we first show how the results change if the securitiser could not commit. We then introduce mortgage servicers into the model and argue that they enable the securitisers to effectively commit to a foreclosure policy. Extending the model to include servicers allows us to derive novel predictions about an endogenous matching between mortgage pools with different quality and servicers with heterogeneous modification capacity, and about servicing contracts with varying biases in foreclosure incentives.

We would like to emphasise that unlike the existenting literature and debates on foreclosure bias, servicers in our model are not subject to moral hazard or other frictions from incomplete contracts, i.e. securitisers can hire them to implement any foreclosure policy without (agency) cost. Instead, they play a signalling role in securitisation and hence the premise of the analysis requires their compensation contracts and loan foreclosure/modification ability to be observable by MBS investors. In practice, servicers' compensations and limitations to loan foreclosure/modification are governed by Pooling and Servicing Agreements (PSA) which are public information and investors are recommended to review them carefully.[24] Also, as Ashcraft and Schuermann (2008) point out, rating agencies recognise the significance of servicers' ability in loan loss mitigation.[25]

4.1 The effect of commitment

In order to formally study the importance of the securitiser's ability to commit to a foreclosure policy, we modify the sequence of events in the model, as shown in Table 2.

Table 2: Model timeline without commitment

$t = 1$	Securitiser observes π_i and offers $\mathcal{F}_i$
$t = 2$	Mortgage defaults in state B Securitiser chooses foreclosure policy λ_i
$t = 3$	Final payoffs realise

The crucial change here is that the securitiser chooses the foreclosure policy *after* she has sold the MBS to the investors. This implies that promised foreclosure policies at $t = 1$ are no longer credible and thus cannot signal any information because the investors anticipate that the securitiser would always choose the foreclosure policy that maximises the value of her residual claim when mortgage defaults

[24] See Kruger (2016), Thompson (2011) and Dechert (2013).

[25] For example, Moody's Investor Service provides Servicer Quality (SQ) Assessments which are "opinions of the ability of a servicer to prevent or mitigate loss in a securitization" (p.26, Moody's (2016)) .

occur at $t = 2$. We defer the characterisation of the securitiser's problem without commitment to the appendix, and discuss the main intuition below.

To demonstrate the incentive problem associated with the lack of commitment, consider the high-type securitiser's incentive to foreclose the delinquent mortgage at $t = 2$ in state B, for a given MBS issued at $t = 1$. Recall that when Eq. 5 holds, an optimal MBS with commitment is a debt security with face value $F_H^* \in (Z_B + \mathcal{L}(\lambda_H^*), Z_B + \mathcal{L}(\lambda_H^*) + (1 - \lambda_H^*)X)$ and the securitiser would like to commit to an excessive foreclosure policy $\lambda_H^* > \lambda^{FB}$ (as described in Proposition 1).

Without commitment, however, this foreclosure policy is not incentive compatible at $t = 2$. Given the MBS with face value F_H^* issued at $t = 1$, the securitiser holds the residual levered-equity claim at $t = 2$, which only pays off in the recovery state. Therefore, instead of implementing the promised foreclosure policy λ_H^*, the securitiser would be better off deviating to a more lenient foreclosure policy. By allowing more loan modification, the securitiser increases the cash flow of the mortgage pool in the recovery state, and thus her expected payoff.

The above reasoning highlights the stark contrast between the securitiser's incentive to foreclose delinquent loans at $t = 1$ and at $t = 2$. The high-type securitiser optimally issues a debt security at $t = 1$ and retains the residual equity stake to signal the quality of her mortgage pool. In order to reduce the retention cost, she would like to commit to an excessive foreclosure policy. Yet precisely because of the retained equity stake necessary as a signal at $t = 1$, the securitiser at $t = 2$ faces distorted incentives, which prevent her from implementing the ex ante desired excessive foreclosure policy.

The lack of commitment thus limits the foreclosure policy the securitiser can credibly choose in equilibrium. This in turn reduces the efficiency of securitisation in equilibrium, as demonstrated by the following proposition. Denote by U_i^{NC} the expected payoff to a securitiser with a type i mortgage pool in the least cost separating equilibrium without commitment, where NC stands for "No Commitment". Proposition 4 establishes $G(\lambda^{FB}) > 0$ as a sufficient condition for the constraint imposed by the lack of commitment to bind and lead to efficiency loss, where $G(\cdot)$ is given by Eq. 5.

Proposition 4. *If $G(\lambda^{FB}) > 0$, compared to the scenario without commitment, the low-type securitiser in the least cost separating equilibrium with commitment is* *equally well off whereas the high-type securitiser is strictly better off. That is,*

$$U_L^{FB} = U_L^* = U_L^{NC} \quad \textit{and} \quad U_H^{FB} > U_H^* > U_H^{NC}$$

4.2 Mortgage servicers enable commitment

The high-type securitiser has a commitment problem because her incentives at $t = 1$ to signal her quality through excessive foreclosure conflicts with her incentives at $t = 2$ to maximise the value of her residual claim. Outsourcing the foreclosure decision to a third-party agent could thus avoid such conflict and hence effectively restore the securitiser's commitment power. The extensions of our model corroborates with some empirical findings regarding the servicers' impact on foreclosures and their compensation contracts.

We propose two potential ways in which the securitiser can commit to a foreclosure policy through contracting with a mortgage servicer. First, if servicers have different mortgage foreclosure capacities or modification capacities and such heterogeneity is public information, the high-type securitiser can signal and effectively commit to a "tough" foreclosure policy by hiring a servicer with high foreclosure capacity (or low modification capacity). This mechanism rests on the assumption that it is costly for servicers to alter their capacity significantly. We believe this assumption is realistic. For example, Thompson (2009), among others, argue that foreclosing and modifying a delinquent mortgage require substantial human capital and expertise hence it is very difficult and costly for a servicer expand the capacity quickly. This interpretation of heterogeneity among mortgage servicers corroborates with the economically and statistically significant servicer fixed effect in predicting foreclosure probability, as emphasised by Agarwal et al. (2011).

Alternatively, the securitiser can hire a servicer and provide him a compensation contract that encourages foreclosure. This can explain why mortgage servicing contracts with biased incentives are offered in practice. Thompson (2009) and Kruger (2016) argue that the biased incentives of the servicers are a key friction causing excessive foreclosures. Contracting with servicers can implement ex post inefficient foreclosures as long as there are frictions preventing the contract from being renegotiated. Empirically Kruger (2016) shows that indeed mortgage servicing contracts are rarely renegotiated, mainly because by law, it requires the consent of the securitiser, the servicer, and the dispersed MBS investors.

We formalise the above ideas in the two extensions below respectively.

4.2.1 Mortgage servicers with heterogeneous foreclosure capacity

In this section, we extend the model to allow the securitiser to choose a mortgage servicer of known foreclosure capacity at $t = 1$. More specifically, there exists a continuum of servicers with different foreclosure capacity $\tau \in (0, 1)$, which is public information. The foreclosure capacity affects the cost of foreclosure. At $t = 2$, if the servicer with capacity τ forecloses a fraction λ of the delinquent mortgages, he incurs a quadratic private cost of $\frac{\kappa}{2}(\lambda - \tau)^2$ for some $\kappa > 0$. The servicer chooses the foreclosure policy at $t = 2$ to minimise the cost. It follows that the servicer chooses to foreclose a fraction $\lambda = \tau$ of the delinquent mortgages. We henceforth refer to τ as the intrinsic "toughness" of the servicer.

At $t = 1$, after learning the quality of her mortgage pool π_i, the securitiser can choose to hire a servicer with capacity τ_i. Each servicer has a reservation utility of 0, and is therefore willing to work for a securitiser if chosen. The securitiser then announces the identity of the servicer alongside the security $\mathcal{F}$ that she offers to the investors. The overall timeline of this extension of the model is summarised in Table 3.

Table 3: Model timeline with heterogeneous servicers

$t = 1$	Securitiser observes π_i Securitiser chooses servicer τ_i and offers $\mathcal{F}_i$
$t = 2$	Mortgage defaults in state B Servicer's foreclosure policy $\lambda_i = \tau_i$
$t = 3$	Final payoffs realise

It is important to notice that the intrinsic "toughness" of the servicers directly affects the subsequent foreclosure policy in equilibrium, because the foreclosure decision is now made by the servicer. Therefore the low-type securitiser would like to choose a "neutral" sevicer with $\tau_L = \lambda^{FB}$ to implement the first-best foreclosure policy, whereas the problem faced by the high-type securitiser at $t = 1$ can be re-written as

$$
\begin{aligned}
&\max_{F_H, \tau_H} \quad p_H(F_H, \lambda_H) + \delta[V_H(\lambda_H) - p_H(F_H, \lambda_H)] \\
s.t \quad &\lambda_H = \tau_H \\
&\text{and } (IC) \text{ as given by Eq. 4}
\end{aligned} \tag{11}
$$

As discussed in Section 3, at $t = 1$, the high-type securitiser benefits from committing to an excessive foreclosure policy. In this extensions, this can be exactly achieved by hiring a servicer who is known to be "tough", as stated below.

Proposition 5. *In the least cost separating equilibrium, the high-type securitiser hires an excessively "tough" servicer, whereas the low-type securitiser hires a "neutral" servicer. That is,* $\tau_H^* = \lambda_H^* \geq \lambda^{FB} = \tau_L^*$. *The equilibrium foreclosure policy is* $(\lambda_H^*, \lambda_L^*)$.

Proposition 5 highlights that, not only do the servicers hired in equilibrium implement the ex ante optimal foreclosure policy, they also serve as a signalling device at the securitisation stage. By hiring an excessively "tough" servicer, the high-type securitiser signals to the investors the quality of her mortgage pool, and reduces the costly retention in equilibrium.

4.2.2 Mortgage servicing contract with biased incentives

In this section, we extend the model to allow the securitiser to explicitly contract with the servicer at $t = 1$. Unlike in the previous subsection, in this extension we assume that the servicer is free to choose any foreclosure policy without incurring any cost, consistent with the setup in the baseline model in which the foreclosure decision is made by the securitiser. This then creates a role for incentive contracts, in order for the securitiser to induce the servicer to make the desired foreclosure policy at $t = 2$.

We assume that at $t = 1$, after learning the quality of her mortgage pool π_i, the securitiser offers a servicing contract to the servicer. A contract (α_i, β_i) specifies a percentage fee to the servicer based on the repayments from the performing (or recovered) mortgages $\alpha_i > 0$, and a percentage fee based on the foreclosure proceeds

$\alpha_i \beta_i \geq 0$.[26] The parameter β_i measures the relative pay from foreclosure compared to modification. The expected payoff to the servicer, given a foreclosure policy of λ, is given by

$$\hat{\pi}\alpha_i \left[\beta_i \mathcal{L}(\lambda) + \theta(1-\lambda)X\right] \tag{12}$$

where $\hat{\pi}$ is the servicer's expectation about the securitiser's type π_i, and plays no role in characterising the equilibrium foreclosure policy. The servicer has a reservation utility of 0, and is therefore willing to accept any servicing contract of non-negative value.

Once accepted by the servicer, the servicing contract (α_i, β_i) is public information at $t = 1$. The securitiser then offers a security $\mathcal{F}_i$ to the investors. The timeline of this version of the model is summarised in Table 4.

Table 4: Model timeline with servicing contracts

$t = 1$	Securitiser observes π_i
	Securitiser offers the servicer a servicing contract (α_i, β_i)
	Securitiser offers investors a security $\mathcal{F}_i$ and discloses (α_i, β_i)
$t = 2$	Mortgage defaults in state B
	Servicer chooses foreclosure policy λ_i
$t = 3$	Final payoffs realise

We now solve for the servicing contracts and foreclosure policy in equilibrium backwards. At $t = 2$, when the mortgages default in the bad state, the servicer chooses the foreclosure policy that maximises his expected payoff as specified by the servicing contract. The resulting foreclosure policy λ_s, where the subscript s stands for "servicer", is characterised by the first order condition $\beta_i \frac{\partial \mathcal{L}(\lambda_s)}{\partial \lambda} - \theta X = 0$. It is useful to notice that the incentive for the servicer to foreclosure the delinquent mortgages is determined by the parameter β_i of the contract, as stated in the following lemma.

[26] This specification of the contract resembles a servicing agreement in practice, and is without loss of generality. Any non-decreasing contract based on the cash flows of the mortgage pool in the bad state can be written as a contract (α, β). We also abstract from payments for servicing the non-defaulting loans, as they play no role in affecting how the servicing contracts affect the foreclosure of delinquent mortgages.

Lemma 4. *For a given servicing contract* (α_i, β_i)*, the foreclosure decision of the servicer at* $t = 2$ *is strictly increasing in* β_i*, and independent of* α_i*. The servicer chooses the first-best foreclosure decision* λ^{FB} *if and only if* $\beta_i = 1$*.*

We will henceforth refer to a contract with $\beta_i = 1$ as an "unbiased" contract, and a contract with $\beta_i > 1$ as a "biased" contract towards foreclosure. Anticipating the incentive effects of the servicing contract, the securitiser chooses the contract as well as the security to offer at $t = 1$, in order to maximise her payoff. We defer the full characterisation of the equilibrium to the appendix, and discuss the main intuition below.

Proposition 6. *In the least cost separating equilibrium, the high-type securitiser provides the servicer with biased incentives towards foreclosure* $\beta_H^* \geq 1$*, whereas the low-type securitiser provides the servicer with unbiased incentives* $\beta_L^* = 1$*. The equilibrium foreclosure policy is* $(\lambda_H^*, \lambda_L^*)$*.*

The low-type securitiser offers an *unbiased* servicing contract to the servicer in equilibrium, in order to implement the first-best foreclosure policy. The high-type securitiser, on the other hand, would prefer an excessive foreclosure policy, because excessive foreclosure reduces the signalling cost she must incur in order to separate from the low type. Therefore, the high-type securitiser optimally chooses to offer a *biased* servicing contract towards foreclosure in equilibrium.

5 Ex ante screening effort and welfare

So far we have treated the ex ante probability γ of the mortgage pool being of high quality as exogenous. In this section, we extend the model to incorporate an ex ante stage $t = 0$, at which time the securitiser can endogenously exert non-verifiable costly effort to increase the probability of receiving a high-quality mortgage pool at $t = 1$. The main finding is that while information asymmetry leads to underinvestment in screening effort, committing to an ex post excessive foreclosure policy mitigates this underinvestment problem and the associated inefficiency.

At $t = 0$, the securitiser is endowed with \$1 and can invest in a mortgage pool. When investing, the securitiser can exerts non-contractible effort to affect $\gamma \in [\underline{\gamma}, \bar{\gamma}]$, the probability that the mortgage pool is of high quality at $t = 1$, where

$0 \leq \underline{\gamma} < \bar{\gamma} \leq 1$. Such effort can be interpreted as, for example, time and resources spent to screen out borrowers with suspicious income or to form mortgage pool with better diversification property. The effort incurs a quadratic cost of $\frac{1}{2}k(\gamma - \underline{\gamma})^2$. We assume $k \geq \frac{U_H^{FB} - U_L^{FB}}{\bar{\gamma} - \underline{\gamma}}$ to guarantee an interior optimal level of effort, and $U_L^{FB} \geq 1$ so that investing in the mortgage pool is always efficient.

5.1 Optimal screening effort

In this section we solve for the optimal screening effort of the securitiser in equilibrium. The securitiser is willing to exert costly effort because the expected payoff to the high type U_H is higher than that to the low type U_L . Since U_H and U_L will be potentially affected by the information environment, the security design, and the foreclosure policy in the subsequent stages of the model, the optimal screening effort chosen by the securitiser will be indirectly affected.

Notice that since the subsequent securitisation stage is in the least cost separating equilibrium, the equilibrium outcome does not depends on γ, the prior probability that the mortgage pool is of high quality. We can therefore consider any generic pair of $\{U_H, U_L\}$ that represents the expected payoffs to the securitiser in the separating equilibrium at the securitsation stage $t = 1$. At $t = 0$, the securitiser chooses the optimal level of effort to maximise her ex ante expected payoff

$$\max_{\gamma} \quad \gamma U_H + (1 - \gamma) U_L - \frac{1}{2}k(\gamma - \underline{\gamma})^2 \tag{13}$$

The optimal effort is thus

$$\gamma^*(U_H, U_L) = \underline{\gamma} + \frac{U_H - U_L}{k} \tag{14}$$

The optimal effort chosen by the securitiser is increasing in the difference in the expected payoff $(U_H - U_L)$ between a high-quality and a low-quality pool. We will look at how this difference changes under symmetric and asymmetric information, and under different foreclosure policy.

First note that the low-type securitiser can always attain the highest possible payoff given her type, i.e. $U_L = U_L^{FB}$, because she suffers no information friction and hence optimally chooses the efficient foreclosure policy λ^{FB} and sells a full pass-through security. On the other hand, the high type is strictly worse off under asymmetric information because of the signalling cost (Proposition 4). As a result, the securitiser exerts *strictly less* effort.

Proposition 7. *Comparing to the symmetric information case, the securitiser expends less screening effort under asymmetric information. The ability to commit to a foreclosure policy at $t = 1$ enhances screening effort at $t = 0$. That is,*

$$\gamma^*(U_H^{FB}, U_L^{FB}) > \gamma^*(U_H^*, U_L^*) \geq \gamma^*(U_H^{NC}, U_L^{NC})$$

where the weak inequality is strict when $G(\lambda^{FB}) > 0$, where $G(\lambda)$ is given by Eq. 5.

As commitment power over foreclosure policy not only improves efficiency (Proposition 4) but also enhances the ex ante screening effort as stated in the above proposition, this result further points to an additional benefit of mortgage servicers as commitment devices.

5.2 Foreclosure policy and screening effort

Next we turn to the question of how regulatory interventions of foreclosure policies can affect the screening effort. As shown by our main result, committing to an excessive foreclosure policy allows the high-type securitiser to reduce the signalling costs incurred in equilibrium. Our extension reveals that this also creates a stronger incentives for the securitiser to screen to create a high-quality mortgage pool, further increasing higher welfare. The following proposition summarises the effect of a regulatory intervention in the foreclosure policy on ex ante screening effort and on welfare.

Proposition 8. *If the government imposes a foreclosure policy λ_H different from the equilibrium policy λ_H^*, including the ex post efficient policy λ^{FB}, the securitiser exerts less screening effort at $t = 0$, hence reducing the total welfare.*

Proposition 8 highlights an unintended consequence of government regulation of the foreclosure decision in the mortgage securitisation market. Due to information asymmetry in securitisation, imposing any foreclosure policy different from λ_H^* on the securitiser *reduces* her payoff in the case of receiving a high-quality mortgage pool. This in turn lowers her incentive to exert screening effort to obtain a high-quality pool. This under-provision of value-enhancing screening effort decreases social welfare.

6 Empirical implications

This section summarises the novel empirical implications of the paper. Our model generates predictions on how mortgage pool quality correlates with, respectively, foreclosure rate, loan losses, and servicers' capacity/incentive to foreclosure mortgages.

6.1 Implications with respect to *unobservable* loan qualities

Our mechanism posits that securitisation under information asymmetry creates a rationale for additional foreclosures in high-quality pool. Therefore the main novel empirical prediction of our model is a *cross-sectional difference-in-difference* predictions with respect to *unobservable* loan qualities as stated below.

Controlling for observable loan characteristics, denote the equilibrium foreclosure policies of high- and low-quality **securitised pools** by $\{\lambda_H^*, \lambda_L^*\}$ and their difference as $\Delta^* \equiv (\lambda_H^* - \lambda_L^*)$. Correspondingly, denote the equilibrium foreclosure policies of high- and low-quality **portfolio loans** by $\{\lambda_H^P, \lambda_L^P\}$ and the difference by $\Delta^P \equiv (\lambda_H^P - \lambda_L^P)$.

Empirical prediction 1. *Controlling for observable loan characteristics, the difference in the foreclosure policy between a high-quality and a low-quality mortgage pool is greater for securitised loans than for portfolio loans. That is,*

$$\Delta^* - \Delta^P > 0$$

Similar cross-sectional difference-in-difference predictions apply to other outcome variables including ***loan loss***, ***foreclosure bias in mortgage servicing contract***, *and* ***the servicers' foreclosure capacity***.

What empirical prediction 1 says is that while we acknowledge that in practice there are forces and constraints outside of the model that could drive the difference in foreclosure rate between *unobservably* high-quality and low-quality mortgage pools, such difference should be larger for securitised mortgage pools in the presence of information asymmetry due to the signalling-cost-reduction effect of foreclosure. Because more foreclosure implies i) larger loan losses (Prop. 1), ii) larger foreclosure bias in the servicers compensation contracts (Prop. 6), and iii) matching with servicers' with more foreclosure capacity (Prop. 5), similar predictions apply.

An econometrician's challenge is to observe the banks' private information on the quality of securistised pools and portfolio loans in order to estimate the different foreclosure likelihood.[27] For example, in a study of syndicated loans, Balasubramanyan et al. (2016) uses banks' internal rating from the Shared National Credit (SNC) dataset as a proxy for banks' private information. For securitised pools, Begley and Purnanandam (2017) and Adelino et al. (2016) proxy the ex ante unobservable pool quality with ex post delinquency rate. After controlling for observable characteristics, a mortgage pool with low ex post delinquency rate would correspond to the high-quality pool in our model.

6.2 Implications with respect to *observable* loan qualities

An interpretation of the model provides further predictions on how the magnitude of the foreclosure bias relates to *publicly observable*, *ex ante* quality of loans. Recall that γ is the prior probability that a securitised mortgage pool is of high quality. We apply the same interpretation of γ to the individual loans and interpret γ as some publicly observable signal of a given loan's quality such as the borrower's FICO score. Specifically, a loan with a higher FICO score is of *observably* higher quality and has a higher probability of being a loan with low credit risk.

For a given observable quality γ, the *average* foreclosure rate for securitised loans and portfolio loans are $\lambda^*(\gamma) = \gamma\lambda^*_H + (1-\gamma)\lambda^*_L$ and $\lambda^P(\gamma) = \gamma\lambda^P_H + (1-\gamma)\lambda^P_L$ respectively.

Empirical prediction 2. *The average foreclosure bias of securitised loans with*

[27] Another challenge is to address the concern that loans are selected into securitised pools by the informed originators/securitisers. Existing empirical works including Piskorski et al. (2010), Agarwal et al. (2011), Kruger (2016) have dealt with the selection bias with various strategies.

respect to portfolio loans is larger for ex ante higher quality loans. That is,

$$\frac{\partial(\lambda^*(\gamma) - \lambda^P(\gamma))}{\partial\gamma} = \Delta^* - \Delta^P > 0$$ [28]

Similar cross-sectional difference-in-difference predictions apply to other outcome variables including ***loan loss****,* ***foreclosure bias in mortgage servicing contract****, and* ***the servicers' foreclosure capacity****.*

There is existing evidence of larger foreclosure bias in loans with higher FICO scores. Both Piskorski et al. (2010) (Table 8) and Agarwal et al. (2011) (Table 3, Panel D) divide their sample into three categories by FICO scores (below 620, between 620 and 680, and above 680) and find stronger foreclosure bias for loans with higher FICO scores.

7 Conclusion

This paper studies the relationship between the foreclosure decision of delinquent mortgages and the securitisation of mortgages. We propose a novel mechanism in which excessive foreclosure policies, in addition to the retention of junior securities, serves as costly signals to reduce informational frictions inherent in the securitisation process. Our theoretical analysis highlights the importance of security design for the foreclosure policy. We list the empirical predictions coming from our model, some of which explain several important observed patterns and empirical findings in the mortgage securitisation industry.

Our paper also suggests that mortgage servicers could have the important role as commitment devices, allowing the securitiser to optimally commit to ex post excessive foreclosure policies. As a result, the mortgage servicing contracts appear to have biased incentives towards foreclosure, and the servicer-specific capacity related to foreclosure can be informative about the quality of the underlying mortgage

[28] It follows from a direct derivation:

$$\begin{aligned}\frac{\partial(\lambda^*(\gamma) - \lambda^P(\gamma))}{\partial\gamma} &= \frac{\partial}{\partial\gamma}(\lambda^*_L + \gamma(\lambda^*_H - \lambda^*_L) - \lambda^P_L - \gamma(\lambda^P_H - \lambda^P_L)) \\ &= \frac{\partial}{\partial\gamma}(\lambda^*_L - \lambda^P_L + \gamma(\Delta^* - \Delta^P)) = \Delta^* - \Delta^P > 0\end{aligned}$$

pool. These results are broadly consistent with empirical findings and yield new predictions for future empirical work.

For a normative perspective, our results caution that policies attempting to restore ex post foreclosure efficiency can have the unintended consequence of reducing the securitisers' ex ante screening effort, thereby worsening the average quality of the mortgage pools and reducing social welfare.

We conclude with some conjecture of directions for future work and extensions. First, this framework can be extended to a setting with multiple securitisers to study the spillover effects of foreclosure. For instance, it would be interesting to study the interaction between the excessive foreclosure policies due to securitisation and the fire-sale externality in the distressed property market. It could also be fruitful to analyse, in a general equilibrium, the potential impact of securitisation on the quantity, quality, and the prices of mortgages originated. Finally, a dynamic framework could shed light on how the excessive foreclosure due to securitisation interacts with property prices across business cycles.

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Appendices

A Proofs

A.1 Proof of Lemma 1

This result follows immediately from the discussion.

A.2 Proof of Lemma 2

We will first show no pooling PBE satisfy the Intuitive Criterion. And then we show the same for any separating PBE other than the least cost separating PBE.

The logic of the proof is as follows: for any candidate pooling PBE (U_H^P, U_L^P) with an offer $\{\mathcal{F}^P, \lambda^P\}$, we construct an off-equilibrium pooling offer $\{\mathcal{F}', \lambda^P\}$ that prunes the candidate PBE with Intuitive Criterion. Since we do not involve changing λ^P in the following analysis, for the ease of notation we will simply denote an offer with $\mathcal{F}$ whenever it does not create confusion.

We begin by applying the Intuitive Criterion to our two-type model as follows: a PBE fails to satisfy the Intuitive Criterion if there exists an unsent offer $\mathcal{F}'$, such that the type H is strictly better off than at the posited PBE by proposing $\mathcal{F}'$ for all best responses with beliefs focused on H, and the type L is strictly better at the posited PBE than at $\mathcal{F}'$ for all best responses for all beliefs in response to $\mathcal{F}'$.

Define $J_H(\mathcal{F}')$ and $J_L(\mathcal{F}')$ as the payoff of the H and L type when they deviate to the off-equilibrium offer $\mathcal{F}'$ under a belief focused on H

$$
\begin{aligned}
J_H(\mathcal{F}') &\equiv p_H(\mathcal{F}') + \delta[V_H - p_H(\mathcal{F}')] \\
J_L(\mathcal{F}') &\equiv p_H(\mathcal{F}') + \delta[V_L - p_L(\mathcal{F}')]
\end{aligned} \tag{15}
$$

Therefore a pooling PBE (U_H^P, U_L^P) does not satisfy the intuitive criterion if there exists an $\mathcal{F}'$ such that $J_H(\mathcal{F}') > U_H^P$ and $J_L(\mathcal{F}') < U_L^P$.

We begin the proof with establishing some useful properties of any pooling PBE

(U_H^P, U_L^P). First, the payoffs can be computed as follows:

$$U_H^P \equiv \bar{p}(\mathcal{F}^P) + \delta[V_H - p_H(\mathcal{F}^P)]$$
$$U_L^P \equiv \bar{p}(\mathcal{F}^P) + \delta[V_L - p_L(\mathcal{F}^P)] \tag{16}$$

where $\bar{p}(\mathcal{F}) = \bar{\pi} f_1 + (1-\bar{\pi})[\theta f_2 + (1-\theta) f_3)]$ and $\bar{\pi} \equiv \gamma \pi_H + (1-\gamma)\pi_L$.

Second, in any pooling PBE that satisfies Intuitive Criterion, both types must attain weakly higher payoffs than the least cost separating (LCS) payoffs (U_H^*, U_L^*). The following claim establishes this property formally.

Claim 1. *For any pooling PBE (U_H^P, U_L^P) that satisfies the Intuitive Criterion, $U_H^P \geq U_H^*$ and $U_L^P \geq U_L^*$.*

Proof. This claim is proved by contradiction. First of all, $U_L^P < U_L^*$ cannot be a PBE because the low type can always attain at least the LCS payoffs U_L^* by deviating to the first-best offer of the low type.

Suppose now $U_H^P < U_H^*$ and $U_L^P \geq U_L^*$. To invoke the Intuitive Criterion, consider a set of beliefs that all deviations are done by the high type. Then by deviating to (F_H^*, λ_H^*), the high type achieves her LCS payoff $U_H^* > U_H^P$ whereas the low type's payoff $p_H(F_H^*, \lambda_H^*) + \delta[V_L(\lambda_H^*) - p_L(F_H^*, \lambda_H^*)]$, is also equal to her LCS payoff U_L^* because (F_H^*, λ_H^*) is the solution of the LCS problem in Eq. 4 and the (IC) therein is binding at the solution. Now consider another offer $\{F', \lambda_H^*\}$ with $F' = F_H^* - \epsilon$ for some arbitrarily small and positive ϵ such that the high type's payoff with this off-equilibrium offer is $U'_H \in (U_H^P, U_H^*)$. Such an F' exists because $U_H^P < U_H^*$ and $F_H^* > c_3$ (Lemma 3). Finally the low type's payoff with the offer $\{F', \lambda_H^*\}$ is $U'_L < U_L^* \leq U_L^P$. □

The third property is shown in the following claim

Claim 2. *In any pooling PBE with offer $\{\mathcal{F}^P, \lambda^P\}$, $f_1^p > c_3(\lambda^P)$.*

Proof. Suppose instead $f_1^P \leq c_3(\lambda^P)$. Because of (MNO), $c_3 \geq f_1^P \geq f_2^P \geq f_3^P$

$$U_L^P \leq \delta V_L(\lambda^P) + (1-\delta)c_3(\lambda^P) < V_L(\lambda^P) \leq V_L(\lambda^{FB}) \equiv U_L^*$$

which contradicts the fact that $U_L^P \geq U_L^*$. □

We are now equipped to construct the PBE pruning offer $\mathcal{F}'$ for any pooling PBE with offer $\mathcal{F}^P$. First, we parametrise a series of offers with y such that

$$\mathcal{F}(y) = \{f_1^P - y, f_2^P - \max\{y - (f_1^P - f_2^P), 0\}, f_3^P\} \tag{17}$$

for $y \in [0, f_1^P - f_3^P]$. Note that $\mathcal{F}(0) = \mathcal{F}^P$ and the domain of y is non-empty thanks to Claim 2 and $f_3^P \leq c_3$ due to limited liability. The rest of the proof involves two claims with the parametrised offer $\mathcal{F}(y)$.

Claim 3. *There exists a unique $\tilde{y} \in (0, f_1^P - f_3^P)$ that satisfies $J_L(\mathcal{F}(\tilde{y})) = U_L^P$*

Proof. The proof is based on the Intermediate Value Theorem. First, $J_L(\mathcal{F}(\epsilon)) > U_L^P$ with $\epsilon \to 0$ because

$$\begin{aligned} J_L(\mathcal{F}(\epsilon)) - U_L^P &= p_H(\mathcal{F}(\epsilon)) - \bar{p}(\mathcal{F}^P) - \delta[p_L(\mathcal{F}(\epsilon)) - p_L(\mathcal{F}^P)] \\ &= p_H(\mathcal{F}^P) - \bar{p}(\mathcal{F}^P) > 0 \qquad \text{as } \epsilon \to 0 \end{aligned}$$

Second, $J_L(\mathcal{F}(f_1^P - f_3^P)) < U_L^P$ as $\mathcal{F}(f_1^P - f_3^P) = \{f_3^P, f_3^P, f_3^P\}$, $f_3^P \leq c_3$ due to (LL), and following the same argument as in Claim 2,

$$J_L(\mathcal{F}(f_1^P - f_3^P)) \leq \delta V_L + (1-\delta)c_3 < V_L(\lambda^P) \leq V_L(\lambda^{FB}) = U_L^* \leq U_L^P$$

Finally, $J_L(\mathcal{F}(y))$ is strictly decreasing and continuous in y

$$\frac{\partial J_L(\mathcal{F}(y))}{\partial y} = \begin{cases} -\pi_H + \delta\pi_L < 0 & \text{for } y \in [0, f_1^P - f_2^P) \\ (1-\theta)(\delta\pi_L - \pi_H) - \theta(1-\delta) < 0 & \text{for } y \in [f_1^P - f_2^P, f_1^P - f_3^P) \end{cases} \tag{18}$$

Therefore, the Intermediate Value Theorem applies. □

Claim 4. $J_H(\mathcal{F}(\tilde{y})) > U_H^P$

Proof. This result relies on two properties:

(i) $J_H(\mathcal{F}(\epsilon)) - U_H^P = J_L(\mathcal{F}(\epsilon)) - U_L^P = p_H(\mathcal{F}^P) - \bar{p}(\mathcal{F}^P) > 0$ as $\epsilon \to 0$;

(ii) $0 > \dfrac{\partial J_H(\mathcal{F}(y))}{\partial y} > \dfrac{\partial J_L(\mathcal{F}(y))}{\partial y}$ for $y \in [0, f_1^P - f_3^P]$

(i) is immediate from the definition of J_H while (ii) from the direct comparison between Eq. 18 and

$$\frac{\partial J_H(\mathcal{F}(y))}{\partial y} = \begin{cases} -\pi_H + \delta\pi_H < 0 & \text{for } y \in [0, f_1^P - f_2^P) \\ (1-\theta)(\delta\pi_H - \pi_H) - \theta(1-\delta) < 0 & \text{for } y \in [f_1^P - f_2^P, f_1^P - f_3^P) \end{cases} \tag{19}$$

These two properties imply that the wedges $J_H - U_H^P$ and $J_L - U_L^P$ are the same when y is arbitrarily close to zero. As y increases, J_L decreases strictly faster then J_H . Therefore, at $\tilde{y}$, the wedge of $J_L - U_L^P$ is zero while the wedge $J_H - U_H^P$ is strictly positive. □

The last step of constructing the PBE pruning $\mathcal{F}'$ is to set $\mathcal{F}' = \mathcal{F}(\tilde{y} + \epsilon_y)$ with an arbitrarily small but positive ϵ_y such that $J_H(\mathcal{F}') > U_H^P$. This ϵ_y exists because $J_H(\mathcal{F}(\tilde{y})) > U_H^P$ as in Lemma 4. And by the properties of $\tilde{y}$ in Lemma 3 and J_L, $J_L(\mathcal{F}') < J_L(\mathcal{F}(\tilde{y})) = U_L^P$. As a result, the posited pooling PBE (U_H^P, U_L^P) cannot satisfy the Intuitive Criterion.

The proof for showing that no separating PBE other than the LCS PBE can satisfy Intuitive Criterion is very similar to Claim 1. Consider a separating PBE (U_H, U_L), by definition of LCS, $U_H \leq U_H^*$ and $U_L \leq U_L^*$ with at least one strict inequality. First U_L cannot be strictly less than U_L^* because the low type can always achieve at least U_L^* by giving the first-best offer. The relevant class of separating PBE is thus with $U_H < U_H^*$ and $U_L = U_L^*$. The remaining argument of the proof follows exactly the same as the one in Claim 1 and therefore is omitted.

A.3 Proof of Lemma 3

In order to establish this result, we first characterise fully the least cost separating equilibrium with security design for the high-type securitiser.

Consider any security $\mathcal{F} = (f_1, f_2(c_2), f_3(c_3))$, which maps the realisation of the mortgage pool cash flows to a set of payoffs to the outside investors, as summarised in Table 5, for any given foreclosure policy λ. Notice that $f_2(c_2)$ and $f_3(c_3)$ also depend indirectly on the foreclosure policy λ through c_2 and c_3. We have suppressed the dependency on λ for brevity. For brevity we also suppress the dependency of

$f_j(c_j)$ on c_j, $j \in \{2,3\}$, whenever it is clear. The value of the MBS $\mathcal{F}$ backed by a mortgage pool of quality i, given a committed foreclosure policy λ, is thus given by

$$p_i(\mathcal{F}, \lambda) = \pi_i f_1 + (1 - \pi_i)[\theta f_2 + (1 - \theta) f_3] \tag{20}$$

Table 5: Payoffs of a generic security backed by the mortgage pool cash flows

Realisation of cash flow	Security payoff $\mathcal{F}$
$c_1 \equiv Z_G$	f_1
$c_2(\lambda) \equiv Z_B + \mathcal{L}(\lambda) + (1-\lambda)X$	$f_2(c_2)$
$c_3(\lambda) \equiv Z_B + \mathcal{L}(\lambda)$	$f_3(c_3)$

In the least cost separating equilibrium, the high-type securitiser maximises the proceeds from securitisation plus the residual cash flow by choosing the security $\mathcal{F}_H$ to offer and her promised foreclosure policy λ_H, while ensuring that it is not profitable for the low type to deviate and mimic.

$$\begin{aligned}
& max_{(\mathcal{F}_H, \lambda_H)} && p_H(\mathcal{F}_H, \lambda_H) + \delta \left[V_H(\lambda_H) - p_H(\mathcal{F}_H, \lambda_H)\right] \\
s.t. \quad & (IC) && U_L^{FB} \geq p_H(\mathcal{F}, \lambda_H) + \delta \left[V_L(\lambda_H) - p_L(\mathcal{F}_H, \lambda_H)\right] \\
& (MNO) && f_1 \geq f_2 \geq f_3 \geq 0 \; \forall \lambda \in (0,1) \text{ and } \frac{\partial f_j(c_j)}{\partial c_j} \geq 0 \; \forall j \in \{2,3\} \\
& (MNI) && c_1 - f_1 \geq c_2 - f_2 \geq c_3 - f_3 \geq 0 \; \forall \lambda \in (0,1) \text{ and} \\
& && \qquad \frac{\partial}{\partial c_j}\left(c_j - f_j(c_j)\right) \geq 0 \; \forall j \in \{2,3\}
\end{aligned} \tag{21}$$

The optimisation programme given by Eq. 21 takes into account the security design problem. Therefore it has two additional monotonicity constraints regulating the security design when compared to Eq. 4: (MNO), the outside investors' monotonicity constraint, and (MNI), the insider residual claim's monotonicity constraint. These constraints state that, respectively, the payoff of the security and the residual payoff to the securitiser are weakly increasing in the realisation of the cash flows.[29]

[29] We specify the two monotonicity constraints for any generic security $\mathcal{F}$ and for all possible realisation of the cash flows. Since the foreclosure policy λ_H is pre-committed, only three cash flows occur in equilibrium, namely c_1, $c_2(\lambda_H)$ and $c_3(\lambda_H)$. The equilibrium security is uniquely defined for these cash flows that occur in equilibrium. Although the payoff of the optimal security may not be uniquely pinned down for the cash flows associated with off-equilibrium foreclosure policies, this is inconsequential for solving the optimal foreclosure policy.

We can now begin to prove this lemma. The proof is constructed by establishing several claims in succession. For any given λ_H, an optimal security maximises the high-type securitiser's expected payoff

$$\delta V_H(\lambda_H) + (1-\delta)p_H(\mathcal{F}_H, \lambda_H)$$

subject to the constrains (IC), (MNO) and (MNI). Since $V_H(\lambda_H)$ is not affected by the security design, the security maximises the selling proceeds $p_H(\mathcal{F}_H, \lambda_H) = \pi_H f_1 + (1-\pi_H)[\theta f_2 + (1-\theta) f_3]$. Since λ_H plays no role in this proof, we subsequently denote proceeds from the sales of the MBS by $p_H(\mathcal{F}_H)$ for the ease of notation.

Given a committed foreclosure policy λ_H, there can only be three cash flow realisations c_1, $c_2(\lambda_H)$, and $c_3(\lambda_H)$ in equilibrium. Denote by f_1^*, f_2^*, and f_3^* the payoffs of the optimal security for these equilibrium cash flow realisations respectively. Claim 5–8 below aim to establish the properties that the equilibrium payoffs of the optimal security must satisfy. We finally characterise the properties of the full security and show that a risky debt as described in Lemma 3 is indeed an optimal security.

Claim 5. *For an optimal security $\mathcal{F}_H^*$, $f_1^* < c_1$.*

Proof. If $f_1^* = c_1$, by (MCI), $f_2^* = c_2(\lambda_H)$ and $f_3^* = c_3(\lambda_H)$. This security (full equity) violates (IC). □

Claim 6. *For any optimal security $\mathcal{F}_H^*$, the (IC) must bind.*

Proof. Suppose instead the (IC) is slack for some optimal security with payoffs $\{f_1^*, f_2^*, f_3^*\}$. By Claim 5, $f_1^* < c_1$. Unless $c_1 - f_1^* = c_2(\lambda_H) - f_2^*$, there exists a security $\hat{\mathcal{F}}$ with payoffs $\{\hat{f}_1, f_2^*, f_3^*\}$ with $\hat{f}_1 > f_1^*$ that satisfies the (IC). As $p_H(\cdot)$ strictly increases with f_1, $p_H(\hat{\mathcal{F}}) > p_H(\mathcal{F}_H^*)$, contradicting the supposition that the security is optimal.

If $f_1^* < c_1$ and $c_1 - f_1^* = c_2(\lambda_H) - f_2^*$, one can increase the objective function $p_H(\cdot)$ by increasing both f_1^* and f_2^* by some $\epsilon > 0$ without violating the (IC), unless $f_2^* = c_2(\lambda_H)$ or $c_2(\lambda_H) - f_2^* = c_3(\lambda_H) - f_3^*$. Note that $f_2^* = c_2(\lambda_H)$ implies $f_1^* = c_1$ hence violates Claim 5.

Suppose now $f_1^* < c_1$ and $c_1 - f_1^* = c_2(\lambda_H) - f_2^* = c_3(\lambda_H) - f_3^*$, similarly one can increase all f_1^*, f_2^*, f_3^* without violating the (IC) to strictly increase $p_H(\cdot)$, unless $f_3^* = c_3(\lambda_H)$. And $f_3^* = c_3(\lambda_H)$ implies $f_1^* = c_1$ hence violates Claim 5.

Since we have shown that any security with a slack (IC) can be improved upon, the (IC) must be binding at any optimal security. $\square$

Claim 7. *For any optimal security $\mathcal{F}_H^*$, $f_1^* > c_3(\lambda_H)$.*

Proof. Suppose instead that $f_1^* \leq c_3(\lambda_H)$. By (MNO), $c_3(\lambda_H) \geq f_1^* \geq f_2^* \geq f_3^*$. This implies that the (IC) is slack because the mimicking payoff

$$\delta V_L(\lambda_H)+p_H(\mathcal{F}_H^*)-\delta p_L(\mathcal{F}_H^*) \leq \delta V_L(\lambda_H)+(1-\delta)c_3(\lambda_H) < V_L(\lambda_H) \leq V_L(\lambda^{FB}) = U_L^*$$

By Claim 6, a slack (IC) contradicts the optimality of $\mathcal{F}_H^*$. $\square$

Claim 8. *Any optimal security $\mathcal{F}_H^*$ has either*

1. $f_1^* = f_2^* > f_3^* = c_3(\lambda_H)$ *or*
2. $f_1^* > f_2^* = c_2(\lambda_H) > f_3^* = c_3(\lambda_H)$

Proof. Consider a security that pays off $\hat{f}_1$, $\hat{f}_2$, and $\hat{f}_3$ for cash flows c_1, $c_2(\lambda_H)$ and $c_3(\lambda_H)$ respectively, such that with the (IC) binds. Using the (IC), write $\hat{f}_1$ as a function of $\hat{f}_2$ and $\hat{f}_3$

$$\hat{f}_1(\hat{f}_2,\hat{f}_3) = \frac{(1-\delta)U_L^* - [(1-\pi_H)-\delta(1-\pi_L)](\theta\hat{f}_2+(1-\theta)\hat{f}_3)}{\pi_H-\delta\pi_L} \tag{22}$$

Substitute this $\hat{f}_1$ into the objective function. After some algebraic manipulation, the objective function becomes

$$\delta V_H + (1-\delta)\left[\frac{\pi_H}{\pi_H-\delta\pi_L}(1-\delta)U_L^* + \delta\frac{\pi_H-\pi_L}{\pi_H-\delta\pi_L}(\theta\hat{f}_2+(1-\theta)\hat{f}_3)\right] \tag{23}$$

which is strictly increasing in $\hat{f}_2$ and $\hat{f}_3$. Since $\hat{f}_2$ is bounded above by either $c_2(\lambda_H)$ or $\hat{f}_1$, and $\hat{f}_3$ only by $c_3(\lambda_H)$, any optimal security $\mathcal{F}_H^*$ must have $f_3^* = c_3(\lambda_H)$ and $f_2^* = \min\{f_1^*, c_2(\lambda_H)\}$. Finally, by Claim 7, $f_1^* > c_3(\lambda_H)$ and hence $f_2^* > c_3(\lambda_H)$. $\square$

Having now analysed the properties of an optimal security's equilibrium payoffs $\{f_1^*, f_2^*, f_3^*\}$, we now consider the security's payoffs associated with the off-equilibrium cash flow realisations, i.e. $f_2(c_2)$ and $f_3(c_3)$ $\forall\lambda \in (0,1)$.

Claim 9. *For any optimal security $\mathcal{F}_H^*$, $f_3(c_3) = c_3(\lambda)\ \forall \lambda \leq \lambda_H$, and either*

1. $f_1^* = f_2^* = f_2(c_2(\lambda))\ \forall \lambda \leq \lambda_H$, *or*

2. $f_1^* > f_2(c_2) = c_2(\lambda)$ *and* $f_3(c_3) = c_3(\lambda)\ \forall \lambda \geq \lambda_H$

Proof. Notice that these payoffs do not affect either the objective function or the (IC). Therefore they are only restricted by the (MNO) and the (MNI). By Claim 8, $f_3^* = c_3(\lambda_H)$. The (MNI) thus implies that $f_3(c_3) = c_3(\lambda)\ \forall \lambda \leq \lambda_H$, because $c_3(\lambda)$ is increasing in λ.

By Claim 8, there are two cases. In the first case, $f_1^* = f_2^*$. The (MNO) then implies that $f_1^* = f_2^* = f_2(c_2(\lambda))\ \forall \lambda \leq \lambda_H$, because $c_2(\lambda)$ is decreasing in λ. In the second case, $f_2^* = c_2(\lambda_H) > f_3^* = c_3(\lambda_H)$. The (MNI) then implies that $f_2(c_2) = c_2(\lambda)$ and $f_3(c_3) = c_3(\lambda)\ \forall \lambda \geq \lambda_H$. □

Finally we can now verify that a risky debt with face value $F_H \in (c_3(\lambda_H), c_1)$, as defined in Lemma 3, indeed is an optimal security as it satisfies Claim 5–9.

A.4 Proof of Proposition 1

The least cost separating equilibrium is characterised by Eq. 4. We prove this proposition by solving the optimisation programme and then highlighting the properties of the equilibrium foreclosure policy.

Firstly, we establish that any optimiser of the programme must bind the (IC). We prove this by contradiction. Suppose there exists (F_H, λ_H) that is an optimiser of the programme such that the (IC) is slack. Then there exists $F'_H > F_H$ such that the (IC) is still satisfied at (F'_H, λ_H). However, the objective function is strictly greater at (F'_H, λ_H) than at (F_H, λ_H). This contradicts with the supposition that (F_H, λ_H) is an optimiser of the programme. Therefore any optimiser of the programme must bind the (IC).

We then substitute the binding (IC) into the objective function to eliminate F_H, and solve the resulting univariate optimisation problem. Let $\hat{F}_H(\lambda_H)$ denote the F_H implied by a binding (IC). Let $u(\lambda_H)$ denote the objective function of the resulting univariate optimisation problem. The solution to the problem characterised by Eq.

4 is equal to $\lambda_H^* = \arg\max_{\lambda_H} u(\lambda_H)$, where

$$u(\lambda_H) = (1-\delta)p_H(\hat{F}_H(\lambda_H), \lambda_H) + \delta V_H(\lambda_H) \tag{24}$$

There can be two cases:

(i) $\hat{F}_H(\lambda_H) \in [Z_B + \mathcal{L}(\lambda_H) + (1-\lambda_H)X, Z_G)$ if and only if $G(\lambda_H) \leq 0$, or

(ii) $\hat{F}_H(\lambda_H) \in (Z_B + \mathcal{L}(\lambda_H), Z_B + \mathcal{L}(\lambda_H) + (1-\lambda_H)X)$ if and only if $G(\lambda_H) > 0$,

where $G(\lambda)$ is given by Eq. 5

Case (i): $G(\lambda_H) \leq 0$ **and** $\hat{F}_H(\lambda_H) \in [Z_B + \mathcal{L}(\lambda_H) + (1-\lambda_H)X, Z_G)$

In this case, the market value of the high type's security is given by

$$p_H(F_H, \lambda_H) = \pi_H F_H + (1-\pi_H)[Z_B + \mathcal{L}(\lambda_H) + \theta(1-\lambda_H)X] \tag{25}$$

A binding (IC) implies that

$$\hat{F}_H(\lambda_H) = \frac{U_L^* - \delta\pi_L Z_G - (1-\pi_H)[Z_B + \mathcal{L}(\lambda_H) + \theta(1-\lambda_H)X]}{\pi_H - \delta\pi_L} \tag{26}$$

We now show that, the objective function of the resulting univariate optimisation programme, $u(\lambda_H)$, is increasing in λ_H if and only if $\lambda_H \leq \lambda^{FB}$. To see this, we differentiate $u(\lambda_H)$ w.r.t. λ_H:

$$\frac{\partial u(\lambda_H)}{\partial \lambda_H} = (1-\delta)\left[\frac{\partial p_H(\hat{F}_H(\lambda_H), \lambda_H)}{\partial \lambda_H} + \frac{\partial p_H(\hat{F}_H(\lambda_H), \lambda_H)}{\partial F_H}\frac{\partial \hat{F}_H(\lambda_H)}{\partial \lambda_H}\right] + \delta\frac{\partial V_H(\lambda_H)}{\partial \lambda_H} \tag{27}$$

Notice that $\frac{\partial u(\lambda_H^{FB})}{\partial \lambda_H} = 0$ because $\frac{\partial p(F_H, \lambda_H^{FB})}{\partial \lambda_H} = 0$ and $\frac{\partial \hat{F}_H(\lambda^{FB})}{\partial \lambda_H} = 0$. Moreover, $u(\lambda_H)$ is strictly concave in λ_H. After some algebraic manipulation, we have

$$\frac{\partial^2 u(\lambda_H)}{\partial \lambda_H^2} = \frac{\delta(1-\pi_H)(\pi_H - \pi_L)}{\pi_H - \delta\pi_L}\mathcal{L}''(\lambda_H) < 0 \tag{28}$$

Therefore, for all λ_H such that $G(\lambda_H) \leq 0$, $u(\lambda_H)$ is increasing in λ_H if and only if $\lambda_H \leq \lambda^{FB}$.

Case (ii): $G(\lambda_H) > 0$ **and** $\hat{F}_H(\lambda_H) \in (Z_B + \mathcal{L}(\lambda_H), Z_B + \mathcal{L}(\lambda_H) + (1-\lambda_H)X)$

In this case, the market value of the high type's security is given by

$$p_H(F_H, \lambda_H) = [\pi_H + (1-\pi_H)\theta]F_H + (1-\pi_H)(1-\theta)[Z_B + \mathcal{L}(\lambda_H)] \tag{29}$$

A binding (IC) implies that

$$\hat{F}_H(\lambda_H) = \frac{U_L^* - \delta\pi_L Z_G - \delta(1-\pi_L)\theta[Z_B + \mathcal{L}(\lambda_H) + (1-\lambda_H)X] - (1-\pi_H)(1-\theta)[Z_B + \mathcal{L}(\lambda_H)]}{[\pi_H + (1-\pi_H)\theta] - \delta[\pi_L + (1-\pi_L)\theta]} \tag{30}$$

We now show that, there exists $\tilde{\lambda}_H \in (\lambda^{FB}, 1]$, such that $u(\lambda_H)$ is increasing in λ_H if and only if $\lambda_H \leq \tilde{\lambda}_H$. To see this, we evaluate the first derivative of $u(\lambda_H)$ w.r.t. λ_H, given by Eq. 27 using Eq 29–30:

$$\begin{aligned}\frac{\partial u(\lambda_H)}{\partial \lambda_H} = (1-\delta)&\frac{[\pi_H + (1-\pi_H)\theta]\,[-\delta(1-\pi_L)\theta(\mathcal{L}'(\lambda_H) - X) - (1-\pi_H)(1-\theta)\,\mathcal{L}'(\lambda_H)]}{[\pi_H + (1-\pi_H)\theta] - \delta[\pi_L + (1-\pi_L)\theta]} \\ &+ (1-\delta)(1-\pi_H)(1-\theta)\,\mathcal{L}'(\lambda_H) + \delta(1-\pi_H)(\mathcal{L}'(\lambda_H) - \theta X)\end{aligned}$$

And the second derivative is given by

$$\begin{aligned}\frac{\partial^2 u(\lambda_H)}{\partial \lambda_H^2} &= [(1-\pi_H)(1-\theta) - \theta(1-\delta)] \\ &\qquad \times \frac{\delta(\pi_H - \pi_L)}{[\pi_H + (1-\pi_H)\theta] - \delta[\pi_L + (1-\pi_L)\theta]}\mathcal{L}''(\lambda_H)\end{aligned} \tag{31}$$

Notice that, depending on the sign of $[(1-\pi_H)(1-\theta) - \theta(1-\delta)]$, $u(\lambda_H)$ can be either concave or convex.

Suppose $[(1-\pi_H)(1-\theta) - \theta(1-\delta)] \leq 0$, then $u(\lambda_H)$ is convex in λ_H. This implies that $u(\lambda_H)$ is increasing in λ_H for all λ_H, as

$$\begin{aligned}\frac{\partial u(\lambda_H)}{\partial \lambda_H} &> -(1-\delta)\frac{[\pi_H + (1-\pi_H)\theta](1-\pi_H)(1-\theta)X}{[\pi_H + (1-\pi_H)\theta] - \delta[\pi_L + (1-\pi_L)\theta]} + (1-\pi_H)(1-\theta)X \\ &= \delta\frac{[\pi_H + (1-\pi_H)\theta] - [\pi_L + (1-\pi_L)\theta]}{[\pi_H + (1-\pi_H)\theta] - \delta[\pi_L + (1-\pi_L)\theta]}(1-\pi_H)(1-\theta)X > 0\end{aligned}$$

where we have used the fact that $\mathcal{L}'(\lambda_H) < X$ for all λ_H (Assumption 1) when deriving the first line.

Suppose $[(1-\pi_H)(1-\theta)-\theta(1-\delta)] > 0$, then $u(\lambda_H)$ is concave in λ_H. At $\lambda_H = \lambda^{FB}$, after some algebraic manipulation using the fact that $\mathcal{L}'(\lambda^{FB}) = \theta X$, we have

$$\frac{\partial u(\lambda^{FB})}{\partial \lambda_H} = \frac{(1-\delta)\delta(\pi_H - \pi_L)\theta(1-\theta)X}{[\pi_H + (1-\pi_H)\theta] - \delta[\pi_L + (1-\pi_L)\theta]} > 0 \tag{32}$$

Therefore there exists $\tilde{\lambda}_H \in (\lambda^{FB}, 1]$, such that $u(\lambda_H)$ is increasing in λ_H if and only if $\lambda \leq \tilde{\lambda}_H$, where $\tilde{\lambda}_H$ is given by $\frac{\partial u(\tilde{\lambda}_H)}{\partial \lambda_H} = 0$ if $\frac{\partial u(1)}{\partial \lambda_H} \leq 0$, and $\tilde{\lambda}_H = 1$ otherwise.

To summarise, there exists $\tilde{\lambda}_H \in (\lambda^{FB}, 1]$, such that for all λ_H such that $G(\lambda_H) > 0$, $u(\lambda_H)$ is increasing in λ_H if and only if $\lambda_H \leq \tilde{\lambda}_H$. After some algebraic manipulation, $\tilde{\lambda}_H$ can be defined by

$$\tilde{\lambda}_H \begin{cases} \text{is defined by} & \text{if } [(1-\pi_H)(1-\theta)-\theta(1-\delta)] > 0 \\ \qquad \mathcal{L}'(\tilde{\lambda}_H) = \frac{\delta - [\pi_H + (1-\pi_H)\theta]}{(1-\pi_H)(1-\theta)-\theta(1-\delta)}\theta X, & \qquad \text{and } \mathcal{L}'(1) < \frac{\delta - [\pi_H + (1-\pi_H)\theta]}{(1-\pi_H)(1-\theta)-\theta(1-\delta)}\theta X \\ = 1, & \text{otherwise} \end{cases} \tag{33}$$

We can now describe the equilibrium foreclosure policy λ_H^*. Notice that $G(\tilde{\lambda}_H) < G(\lambda^{FB})$, where $G(\lambda)$ is given by Eq. 5. This follows because $G(\lambda)$ is decreasing in λ for $\lambda \geq \lambda^{FB}$. To see this,

$$\frac{\partial G(\lambda)}{\partial \lambda} = (1-\delta\pi_L)[\mathcal{L}'(\lambda) - X] + (1-\pi_H)(1-\theta)X \tag{34}$$

For $\lambda \geq \lambda^{FB}$, $\mathcal{L}'(\lambda) \leq \theta X$, and the above expression is smaller than

$$-(1-\delta\pi_L)(1-\theta)X + (1-\pi_H)(1-\theta)X = (\delta\pi_L - \pi_H)(1-\theta)X < 0 \tag{35}$$

The equilibrium foreclosure policy is thus characterised as follows

1. If $G(\tilde{\lambda}_H) < G(\lambda^{FB}) \leq 0$, then $\lambda_H^* = \lambda^{FB}$.

   This follows because, for $\lambda_H \leq \lambda^{FB}$, $u(\lambda_H)$ is increasing in λ_H in either case; for $\lambda_H \geq \lambda^{FB}$, $G(\lambda_H) < \lambda^{FB} \leq 0$, and $u(\lambda^{FB})$ is decreasing as described by Case (i). Therefore, $u(\lambda_H)$ is maximised at $\lambda_H^* = \lambda^{FB}$.

2. If $G(\tilde{\lambda}_H) \leq 0 < G(\lambda^{FB})$, then $\lambda_H^* = \check{\lambda}_H$, where $\check{\lambda}_H > \lambda^{FB}$ is given by

$G(\breve{\lambda}_H) = 0$.

To see this, notice that $G(\tilde{\lambda}_H) \leq 0 < G(\lambda^{FB})$ and that $G(\lambda)$ is decreasing in λ for $\lambda \geq \lambda^{FB}$ imply that there exists $\breve{\lambda}_H \in (\lambda^{FB}, \tilde{\lambda}_H]$ such that $G(\breve{\lambda}_H) = 0$. Moreover, $G(\lambda_H) > 0$ for $\lambda_H \in (\lambda^{FB}, \breve{\lambda}_H)$ and $G(\lambda_H) < 0$ for $\lambda_H > \breve{\lambda}_H$. The result thus follows because, for $\lambda_H \leq \lambda^{FB}$, $u(\lambda_H)$ is increasing in λ_H in either case; for $\lambda_H \in (\lambda^{FB}, \breve{\lambda}_H)$, $G(\lambda_H) > 0$ and $u(\lambda_H)$ is increasing as described by Case (ii); for $\lambda \geq \breve{\lambda}_H > \lambda^{FB}$, $G(\lambda_H) \leq 0$ and $u(\lambda_H)$ is decreasing as described by Case (i). Therefore, $u(\lambda_H)$ is maximised at $\breve{\lambda}_H$.

3. If $0 < G(\tilde{\lambda}_H) < G(\lambda^{FB})$, then $\lambda_H^* = \tilde{\lambda}_H > \lambda^{FB}$, where $\tilde{\lambda}_H$ is given by Eq. 33.

   This follows because, for $\lambda \leq \lambda^{FB}$, $u(\lambda_H)$ is increasing in λ_H in either case; for $\lambda_H \in (\lambda^{FB}, \tilde{\lambda}_H]$, $G(\lambda_H) > 0$ and $u(\lambda_H)$ is increasing in λ_H as described by Case (ii); for $\lambda_H > \tilde{\lambda}_H$, $u(\lambda_H)$ is decreasing in λ_H in either case. Therefore, $u(\lambda_H)$ is maximised at $\tilde{\lambda}_H$.

To summarise, the equilibrium foreclosure policy is $\lambda_H^* \geq \lambda^{FB}$, where the inequality is strict if and only if $G(\lambda^{FB}) > 0$.

A.5 Proof of Corollary 1

This result follows from the proof of Proposition 1. Recall that $u(\lambda_H)$ is defined by Eq. 24. Since $\frac{\partial V_H(\lambda^{FB})}{\partial \lambda_H} = 0$, $\frac{dp_H(\hat{F}_H(\lambda^{FB}),\lambda^{FB})}{d\lambda_H} \geq 0$ if and only if $\frac{\partial u(\lambda^{FB})}{\partial \lambda_H} \geq 0$.

As discussed in Appendix A.4, there are three cases.

1. If $G(\tilde{\lambda}_H) < G(\lambda^{FB}) \leq 0$, then $\frac{dp_H(\hat{F}_H(\lambda^{FB}),\lambda^{FB})}{d\lambda_H} = \frac{\partial u(\lambda^{FB})}{\partial \lambda_H} = 0$ as described by Case (i).

2. If $G(\tilde{\lambda}_H) \leq 0 < G(\lambda^{FB})$, then $\frac{dp_H(\hat{F}_H(\lambda^{FB}),\lambda^{FB})}{d\lambda_H} = \frac{\partial u(\lambda^{FB})}{\partial \lambda_H} > 0$ as described by Case (ii).

3. If $0 < G(\tilde{\lambda}_H) < G(\lambda^{FB})$, then $\frac{dp_H(\hat{F}_H(\lambda^{FB}),\lambda^{FB})}{d\lambda_H} = \frac{\partial u(\lambda^{FB})}{\partial \lambda_H} > 0$ as described by Case (ii).

To summarise, $\frac{dp_H(\hat{F}_H(\lambda^{FB}),\lambda^{FB})}{d\lambda_H} \geq 0$, where the inequality is strict if and only if $G(\lambda^{FB}) > 0$.

A.6 Proof of Proposition 2

We characterise the comparative statics for the three cases discussed in Appendix A.4.

1. If $G(\tilde{\lambda}_H) < G(\lambda^{FB}) \leq 0$, $\lambda_H^* = \lambda^{FB}$.

2. If $G(\tilde{\lambda}_H) \leq 0 < G(\lambda^{FB})$, $\lambda_H^* = \breve{\lambda}_H$, where $\breve{\lambda}_H$ is given by $G(\breve{\lambda}_H) = 0$. In this case, $\frac{\partial \lambda_H^*}{\partial \pi_H} > 0$, because $\frac{\partial G(\lambda_H^*)}{\partial \lambda_H} < 0$ and

$$\frac{\partial G(\cdot)}{\partial \pi_H} = (1-\theta)(1-\lambda)X > 0 \tag{36}$$

3. If $0 < G(\tilde{\lambda}_H) < G(\lambda^{FB})$, $\lambda_H^* = \tilde{\lambda}_H$, where $\tilde{\lambda}_H$ is given by Eq. 33. In this case, $\frac{\partial \lambda_H^*}{\partial \pi_H} > 0$, because the LHS of Eq. 33 is decreasing in λ_H, and the RHS of Eq. 33 is decreasing in π_H. The derivative of the RHS of Eq. 33 w.r.t π_H is equal to

$$-\frac{1-[\delta+(1-\delta)\theta]}{[(1-\pi_H)(1-\theta)-(1-\delta)\theta]^2} < 0 \tag{37}$$

To summarise, $\frac{\partial \lambda_H^*}{\partial \pi_H} \geq 0$, where the inequality is strict if and only if $G(\lambda^{FB}) > 0$.

A.7 Proof of Proposition 3

As discussed in Section 3.3, in a separating equilibrium, the low-type securitiser sells the entire cash flow of the mortgage pool, and chooses the first-best foreclosure policy. The payoff to the low-type securitiser in a separating equilibrium is equal to the first-best level, U_L^{FB}.

We now consider the foreclosure policy of the high-type securitiser, whose optimisation problem is given by Eq. 10. Because the optimisation programme only depends on λ_H through the expected payoff of the mortgage pool in the bad state. We can express $V_i(\lambda_H)$ (Eq. 2) as $V_i(v(\lambda_H))$, where

$$V_i(v) = \pi_i Z_G + (1-\pi_i)v \tag{38}$$

and $v(\lambda) = Z_B + \mathcal{L}(\lambda) + \theta(1-\lambda)X$ is the expected value of the mortgage pool in the bad state. Choosing a foreclosure policy is thus equivalent to choosing a level of $v(\lambda_H)$, and we can rewrite the high-type securitiser's problem as follows:

$$\begin{aligned} &\max_{(\alpha,v)} \quad [\alpha + \delta(1-\alpha)]\, V_H(v) \\ s.t. \quad &(IC) \quad U_L^{FB} \geq \alpha V_H(v) + \delta(1-\alpha)V_L(v) \\ &\text{and} \quad v \in [\min\{v(0), v(1)\}, v(\lambda^{FB})] \end{aligned} \tag{39}$$

We now show that any optimiser of the programme must bind the (IC). We prove this by contradiction. Suppose there exists (α, v) that is an optimiser of the program such that the (IC) is slack. This implies that either $\alpha < 1$, or $v < v(\lambda^{FB})$, or both. This follows because the (IC) is violated if $\alpha = 1$ and $v = (\lambda^{FB})$, i.e. the RHS of the (IC) is equal to $V_H(\lambda^{FB}) > U_L^{FB}$. There thus exists (α', v'), where $\alpha' \geq \alpha$ and $v' \geq v$, with at least one of the inequalities is strict, such that the (IC) is still satisfied at (α', v'). However, the objective function is strictly greater at (α', v') than at (α, v). This contradicts with the supposition that (α, v) is an optimiser of the programme. Therefore any optimiser of the programme must bind the (IC).

We then substitute the binding (IC) into the objective function to eliminate α. Notice that there exists $\alpha \in (0, 1]$ such that the (IC) binds for all v such that $V_H(v) \geq U_L^{FB}$. Therefore the optimisation programme can be expressed as

$$\begin{aligned} &\max_{v} \quad \left[\delta + (1-\delta)\frac{U_L^{FB} - \delta V_L(v)}{V_H(v) - \delta V_L(v)}\right] V_H(v) \\ s.t. \quad &v \in [\min\{v(0), v(1)\}, v(\lambda^{FB})] \quad \text{and} \quad V_H(v) \geq U_L^{FB} \end{aligned} \tag{40}$$

The first derivative of the above objective function w.r.t. v is equal to

$$\begin{aligned} h(v) = &\left[\delta + (1-\delta)\frac{U_L^{FB} - \delta V_L(v)}{V_H(v) - \delta V_L(v)}\right](1-\pi_H) \\ &+ \frac{(1-\delta)V_H(v)}{V_H(v) - \delta V_L(v)}\left[-\delta(1-\pi_L) - \frac{(U_L^{FB} - \delta V_L(v))[(1-\pi_H) - \delta(1-\pi_L)]}{V_H(v) - \delta V_L(v)}\right] \end{aligned} \tag{41}$$

The second derivative of the objective function w.r.t. v is equal to

$$\begin{aligned}\frac{\partial h(v)}{\partial v} &= \frac{2(1-\delta)(1-\pi_H)}{V_H(v)-\delta V_L(v)}\left[-\delta(1-\pi_L) - \frac{(U_L^{FB}-\delta V_L(v))[(1-\pi_H)-\delta(1-\pi_L)]}{V_H(v)-\delta V_L(v)}\right] \\ &\quad + 2(1-\delta)V_H(v)\left[\frac{\delta(1-\pi_L)[(1-\pi_H)-\delta(1-\pi_L)]}{(V_H(v)-\delta V_L(v))^2}\right. \\ &\qquad\qquad \left. + \frac{(U_L^{FB}-\delta V_L(v))[(1-\pi_H)-\delta(1-\pi_L)]^2}{(V_H(v)-\delta V_L(v))^3}\right] \\ &= \frac{2(1-\delta)}{V_H(v)-\delta V_L(v)}\left[\frac{V_H(v)-U_L^{FB}}{V_H(v)-\delta V_L(v)}\delta(1-\pi_L) + \frac{(U_L^{FB}-\delta V_L(v))(1-\pi_H)}{V_H(v)-\delta V_L(v)}\right] \\ &\quad \times \underbrace{\left[-(1-\pi_H) + [(1-\pi_H)-\delta(1-\pi_L)]\frac{V_H(v)}{V_H(v)-\delta V_L(v)}\right]}_{=-\frac{\delta(\pi_H-\pi_L)Z_G}{V_H(v)-\delta V_L(v)}<0} < 0 \end{aligned} \tag{42}$$

This implies that the objective function is strictly concave in v. Since v is bounded, there exists a unique v^{UE} that maximises the expected payoff to the high-type sensitiser. It then follows that the high-type securitiser's optimal foreclosure policy λ_H^{UE} is given by $v(\lambda_H^{UE}) = v^{UE}$.

Moreover, if $h(v(\lambda^{FB})) \geq 0$, then $v^{UE} = v(\lambda^{FB})$ and $\lambda_H^{UE} = \lambda^{FB}$ is the unique solution. If $h(v(\lambda^{FB})) < 0$ and $h(\max\{v(0), v(1)\}) \geq 0$, then v^{UE} is given by $h(v^{UE}) = 0$, and there exist exactly two solutions to $v(\lambda_H^{UE}) = v^{UE}$, where one is greater than λ^{FB} and the other is smaller than λ^{FB}. Finally, if $h(\max\{v(0), v(1)\}) < 0$ and $h(\min\{v(0), v(1)\}) \geq 0$, then $v^{UE} \in [\min\{v(0), v(1)\}, \max\{v(0), v(1)\})$ and there exists a unique $\lambda_H^{UE} \neq \lambda^{FB}$ such that $v(\lambda_H^{UE}) = v^{UE}$.

A.8 Proof of Proposition 4

In order to establish this result, we first characterise fully the least cost separating equilibrium without commitment. As in the main text, we denote all equilibrium quantities with NC for the case with no commitment.

We again start the analysis with the low-type securitiser. Since the low type issues a full pass-through equity security in a separating equilibrium, she retains no cash flow. Maintaining the assumption that in this case she makes the first-best foreclosure decision to maximise the value of the mortgage pool, $\lambda_L^{NC} = \lambda_L^{FB}$, the payoff to a low-type securitiser is therefore equal to $U_L^{NC} = U_L^{FB}$.

The high-type securitiser chooses an optimal security $\mathcal{F}_H$ at $t = 1$ to maximise her expected payoff, taking into account the subsequent foreclosure policy λ_H^{NC} chosen at $t = 2$ given the security issued. In the least cost separating equilibrium, the high type's problem without commitment is

$$\max_{(\mathcal{F}_H, \lambda_H^{NC})} p_H(\mathcal{F}_H, \lambda_H^{NC}) + \delta[V_H(\lambda_H^{NC}) - p_H(\mathcal{F}_H, \lambda_H^{NC})]$$

$$s.t. \qquad (IC) \quad U_L^{NC} \geq p_H(\mathcal{F}_H, \lambda_H^{NC}) + \delta[V_L(\lambda_H^{NC}) - p_L(\mathcal{F}_H, \lambda_H^{NC})]$$

$$(IC^s) \quad \lambda_H^{NC} = \arg\max_{\lambda} \quad \theta\,[c_2(\lambda) - f_2] + (1-\theta)\,[c_3(\lambda) - f_3]$$

$$(MNO) \text{ and } (MNI) \text{ as given by Eq. 21} \tag{43}$$

Except for the constraint (IC^s), the high-type's problem without commitment has the same objective function and constraints as the problem with commitment in Eq. 21. This additional (IC^s) constraint captures the fact that, the securitiser is not able to pre-commit to a foreclosure policy. Instead, the foreclosure policy λ_H^{NC} is chosen at $t = 2$ to maximise the expected value of her residual claim, given the security $\mathcal{F}_H$ issued. Similarly, should the low-type securitiser mimic the security issued by the high type, she also subsequently chooses the same incentive compatible foreclosure policy λ_H^{NC}. This is reflected in the low type's no-mimicking constraint (IC).

We can now prove the remaining parts of this proposition. $U_H^* \geq U_H^{NC}$ follows immediately from the above observation that the opitmisation problem without commitment is the problem with commitment in Eq. 21 with the additional constraint (IC^s). As such, the solution to the problem without commitment is a feasible offer for the problem with commitment. Therefore, the high type securitiser can achieve at least U_H^{NC} when she can commit to a foreclosure policy.

We finally show that $U_H^* > U_H^{NC}$ if $G(\lambda^{FB}) > 0$, where $G(\lambda)$ is given by Eq. 5. To do this, we make use of some properties of the equilibrium optimal MBS $\mathcal{F}_H^*$ issued by the high-type securitiser established in the proof of Proposition 1. If $G(\lambda^{FB}) > 0$, then $f_1^* = f_2^* = F_H^* < c_2(\lambda_H^*)$ and $\lambda_H^* > \lambda^{FB}$. Claim 9 then implies that $f_2(c_2) = F_H^* \; \forall \lambda \leq \lambda_H$. In this case, however, the offer $(\mathcal{F}_H^*, \lambda_H^*)$ does not satisfies the (IC^s) in the problem without commitment, as a marginal decrease in λ from λ_H^* would increase the securitiser's expected payoff at $t = 2$, given by $\theta[c_2(\lambda_H) - F_H^*]$. As a result, any solution $(\mathcal{F}_H^*, \lambda_H^*)$ of the problem with commitment is not admissible in the more constrained problem without commitment if $G(\lambda^{FB}) > 0$, hence $U_H^{NC} < U_H^*$.

A.9 Proof of Proposition 5

This proposition follows immediately from the discussion.

A.10 Proof of Lemma 4

This lemma follows immediately from examining the property of the first order condition $\beta_i \frac{\partial \mathcal{L}(\lambda_s)}{\partial \lambda} - \theta X = 0$. By implicitly differentiating λ_s w.r.t. β_i we have

$$\frac{\partial \lambda_s}{\partial \beta_i} = -\frac{\frac{\partial \mathcal{L}(\lambda_s)}{\partial \lambda}}{\beta_i \frac{\partial^2 \mathcal{L}(\lambda_s)}{\partial \lambda^2}} > 0 \tag{44}$$

Further, since the first-best foreclosure policy is characterised by $\frac{\partial \mathcal{L}(\lambda^{FB})}{\partial \lambda} = \theta X$, $\lambda_s = \lambda^{FB}$ if and only if $\beta_i = 1$.

A.11 Proof of Proposition 6

We fully characterise the least cost separating equilibrium of this extension and establish the results.

We start the analysis with the low-type securitiser. Given any servicing contract, the low type issues a full pass-through equity security in a separating equilibrium. Therefore she chooses a servicing contract to maximise her payoff

$$\begin{aligned} \max_{(\alpha_L, \beta_L)} \quad & V_L(\lambda_s) - \pi_L \alpha_L \left[\beta_L \mathcal{L}(\lambda_s) + \theta(1-\lambda_s)X\right] \\ s.t. \quad & \hat{\pi} \alpha_L \left[\beta_L \mathcal{L}(\lambda_s) + \theta(1-\lambda_s)X\right] \geq 0 \end{aligned} \tag{45}$$

where λ_s is given by the first order condition $\beta_L \frac{\partial \mathcal{L}(\lambda_s)}{\partial \lambda} - \theta X = 0$. The solution to the above problem is $\beta_L = 1$ and $\alpha_L \to 0$. As a result, the equilibrium foreclosure policy of a low-type mortgage pool is λ^{FB}, and the low-type securitiser receives a payoff is equal to U_L^{FB}.

The high-type securitiser, given a servicing contract, chooses an optimal security $\mathcal{F}_H$ at $t = 1$ to maximise her expected payoff, taking into account the subsequent foreclosure policy induced by the servicing contract. We again restrict attention to a risky debt security with face value $F_H \in (Z_B + \mathcal{L}(\lambda_s), Z_G)$. As shown in Lemma 3, risky debt is indeed the optimal security. The high-type securitiser chooses to offer a servicing contract and the security to maximise the proceeds from securitisation plus the residual cash flow less the fees paid to the servicers.

$$
\begin{aligned}
& max_{F_H,(\alpha_H,\beta_H)} \quad p(F_H, \lambda_H) + \delta\left[\pi_H(Z_G - F_H)\right. \\
& \qquad\qquad\qquad\qquad \left. +(1-\pi_H)\theta \max\{Z_B + \mathcal{L}(\lambda_s) + (1-\lambda_s)X - F_H, 0\}\right] \\
& \qquad\qquad\qquad\qquad - \pi_H \alpha_H [\beta\, \mathcal{L}(\lambda_s) + (1-\lambda_s)X] \\
s.t. \quad & \hat{\pi}\alpha_L\left[\beta_L\, \mathcal{L}(\lambda_s) + \theta(1-\lambda_s)X\right] \geq 0 \\
& (MC) \text{ and } (IC) \text{ are given by Eq. 4} \qquad (46)
\end{aligned}
$$

where λ_s is given by the first order condition $\beta_H \frac{\partial\, \mathcal{L}(\lambda_s)}{\partial \lambda} - \theta X = 0$. The solution to the above problem is $\beta_H \geq 1$ such that $\lambda_s = \lambda_H^* \geq \lambda^{FB}$, and $\alpha_H \to 0$. As a result, the equilibrium foreclosure policy of a high-type mortgage pool is λ_H^*, and the high-type securitiser receives a payoff equal to U_H^*.

A.12 Proof of Proposition 7

This proposition follows immediately from Eq. 14 and Proposition 4.

A.13 Proof of Proposition 8

Denote with $U_i(\lambda)$ the expected payoff obtained by the high-type securitiser in the least cost separating equilibrium, for a given foreclosure policy. In this equilibrium, the high-type securitiser chooses a security to offer at $t = 1$ to maximise her expected payoff, while preventing mimicking from the low type. Formally, $U_i(\lambda)$ is equal to the value of the optimisation programme Eq. 4, given $\lambda_H = \lambda$.

By definition of λ_H^* as the optimiser of Eq. 4 , $U_H(\lambda_H^*) = U_H^* > U_H(\lambda_H)$ for any $\lambda_H \neq \lambda_H^*$. Thus the screening effort γ^* decreases as

$$\gamma^*(U_H(\lambda_H), U_L^{FB}) < \gamma^*(U_H(\lambda_H^*), U_L^{FB}) \quad \forall \lambda_H \neq \lambda_H^*$$

For efficiency, we only need to look at the securitiser's expected payoff as the investors are always indifferent. The expected payoff is lower when λ_H^* is replaced with λ_H, i.e.

$$\begin{aligned}
&\gamma^*(U_H(\lambda_H), U_L^{FB})U_H(\lambda_H) + [1-\gamma^*(U_H(\lambda_H), U_L^{FB})]U_L^{FB} - \frac{1}{2}k\gamma^{*2}(U_H(\lambda_H), U_L^{FB})\\
&< \gamma^*(U_H(\lambda_H), U_L^{FB})U_H(\lambda_H^*) + [1-\gamma^*(U_H(\lambda_H), U_L^{FB})]U_L^{FB} - \frac{1}{2}k\gamma^{*2}(U_H(\lambda_H), U_L^{FB})\\
&\leq \gamma^*(U_H(\lambda_H^*), U_L^{FB})U_H(\lambda_H^*) + [1-\gamma^*(U_H(\lambda_H^*), U_L^{FB})]U_L^{FB} - \frac{1}{2}k\gamma^{*2}(U_H(\lambda_H^*), U_L^{FB})
\end{aligned}$$

The first inequality comes from $U_H(\lambda_H) < U_H(\lambda_H^*)$ and the second weak inequality follows from the definition of optimal γ^*. Finally, λ_H^{FB} is one of the possible $\lambda_H \neq \lambda_H^*$ if and only if $G(\lambda^{FB}) > 0$, where $G(\lambda)$ is given by Eq. 5.